

3 VaR Methodologies

Capital Market Risk Advisors



Variance Covariance

Variance Covariance multiplies market value position exposures by the standard deviation of price changes.



Position Inputs Sensitivity Approach

Position	Delta Equivalent
Interest Rate Position	Dollar Value per Basis Point Change in Rate
Foreign Exchange Equity Commodity	Dollar Value of the Position
Options	Dollar Value Adjusted for Option Delta Plus Gamma Expressed as a Change in Delta

Delta Equivalent Examples

Examples:

\$1MM 10 Year USD Swaps ==> \$760/basis point

1,000 shares IBM stock at \$180/share => $1,000 * \$180$ => \$180,000

ATM options on 1,000 shares of IBM stock; delta .45 =>
 $1,000 * .45 * \$180$ => \$81,000

100 S&P 500 futures => $500 * \$1,350$ => \$675,000



Position Sensitivity Aggregation

Position sensitivities can be added together to represent a portfolio level sensitivity to changes in underlying market prices.

Examples:

\$1MM 10 Year USD Swaps ==> \$760/basis point

\$1MM 5 Year USD Swaps ==> \$430/basis point

\$1MM 2 Year USD Swaps ==> \$185/basis point

Total Swaps Portfolio ==> \$1,375/basis point



Variance Covariance VAR Example

One Asset

Position	Delta Equivalent	One Week Volatility	Risk
1,000 shares IBM Stock @ \$180/Share	$1,000 * \$180 = \$180,000$	10%	$\$180,000 * 10\% = \$18,000$

Variance Covariance VAR Example

Two Assets

Position	Delta Equivalent	One Week Volatility	Risk
1,000 shares IBM Stock @ \$180/Share	1,000 * \$180 = \$180,000	10%	\$180,000 * 10% = \$18,000
\$10mm 10 Yr US Treasury Note	\$760/bp * 10 = \$7,600/bp	10 bps	\$7,600/bp * 10 = \$76,000
Portfolio Risk			\$84,820

$$\sqrt{180,000^2 \cdot 10^2 + 7,600^2 10^2 + 2(180,000 \times 7,600 \times .10 \times 10 \times .4)} = \$84,820$$

Variance Covariance VAR Example

Three Assets

Position	Delta Equivalent	One Week Volatility	Risk
1,000 shares IBM Stock @ \$180/Share	1,000 * \$180 = \$180,000	10%	\$180,000 * 10% = \$18,000
\$10mm 10 Yr US Treasury Note	\$760/bp * 10 = \$7,600/bp	10 bps	\$7,600/bp * 10 = \$76,000
45 3 Month Eurodollar Futures	\$25/bp * 45 = \$1,125/bp	6bps	-\$1,125/bp * 6 = -\$6,750
Portfolio Risk			\$88,875

$$\left(180,000^2 \cdot 10^2 + 7,600^2 10^2 + 1,125^2 6^2 + 2(180,000 \times 7,600 \times .10 \times 10 \times .4) \right. \\ \left. + 2(180,000 \times -1,125 \times .10 \times 6 \times .6) + 2(7,600 \times -1,125 \times 10 \times 6 \times .5) \right)^{\frac{1}{2}} = \$88,875$$

VAR Estimation Period

To estimate longer period VAR from One Day VAR, you multiply the One Day VAR by the square root of the estimation period (expressed as # of days).

Finding 1 Year VAR

$$\begin{aligned} \text{1 Year VAR} &= \text{1 Day VAR} * \text{SQRT}(250) \\ &= \$88,875 * \text{SQRT}(250) \\ &= \$1.41\text{mm} \end{aligned}$$

Time Adjustment

1 week	sqrt(5)
1 month	sqrt(21)
3 month	sqrt(62.5)
1 year	sqrt(250)

Variance Covariance VAR Result

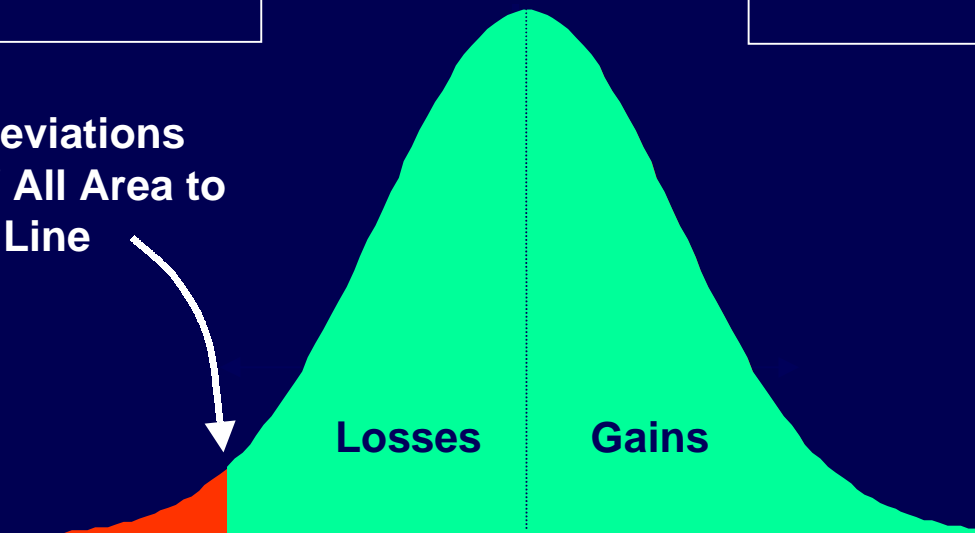
Finding x% Confidence VAR

$$\begin{aligned} 99\% \text{ VAR} &= \text{VAR Result} * 2.33 \\ &= \$1.41 * 2.33 \\ &= \$3.27\text{MM} \end{aligned}$$

Z-Result

<u>Confidence</u>	<u># of σ's</u>
84%	1.00
90%	1.28
95%	1.65
97.5%	1.96
99%	2.33

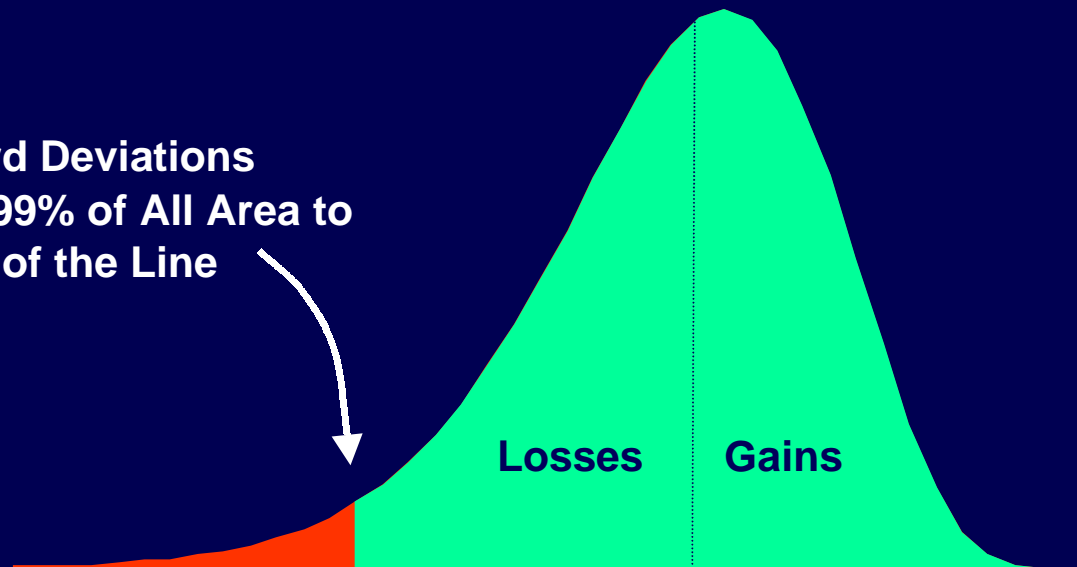
2.33 Standard Deviations
Includes 99% of All Area to
the Right of the Line



Variance Covariance Convexity Adjustment

- ◆ The VAR result can be adjusted to account for some skewness in the distribution caused by the convexity of bonds and options.
- ◆ The adjustment however, is an approximation and does not account for the risks of complex derivative products.

**x Standard Deviations
Includes 99% of All Area to
the Right of the Line**

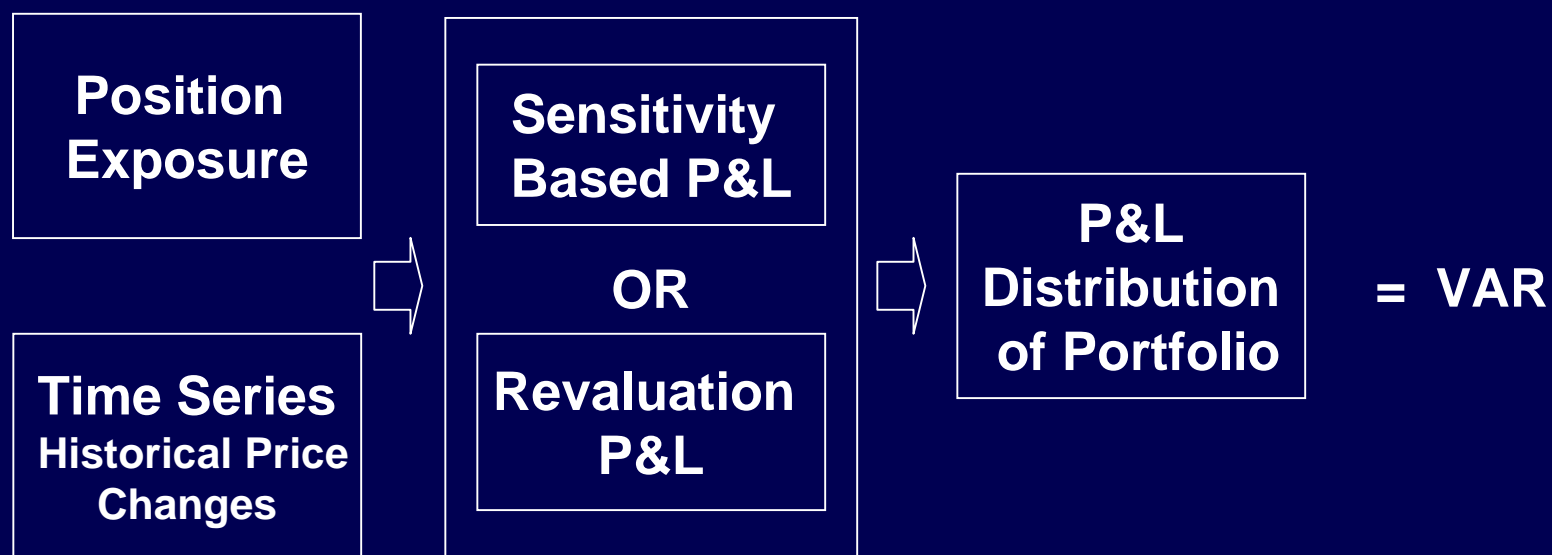


Variance Covariance

Advantages	Disadvantages
<ul style="list-style-type: none">◆ Fast◆ Relatively easy to implement◆ Requires only portfolio level sensitivities◆ Can be modified to capture some measure of convexity◆ Data sets are readily available	<ul style="list-style-type: none">◆ Does not revalue positions◆ Cannot account for complex or discontinuous payoffs◆ Cannot incorporate multiple time horizons◆ Assumes normal or normal-like distributions

Historic Simulation

Historic Simulation calculates the P&L for each day based on observed changes in price.



Historic Simulation Advantages & Disadvantages

Advantages

- ◆ Makes no assumption about distributions (non parametric)
- ◆ Relies on volatility and correlation embedded in time series
- ◆ Captures fat tails (events) in price change distribution

Disadvantages

- ◆ VAR incorporates only historic changes in price
- ◆ Data intensive - needs many time series

Actual Price Changes

Historic Simulation relies on time series of actual price changes - it does not assume a normal distribution.

<u>Date</u>	<u>Rate</u>	<u>Change</u>
Jan2	6.25%	.10
Jan3	6.35%	.10
Jan4	6.40%	.05
Jan5	6.15%	-.25
.	.	.
.	.	.
.	.	.
Dec31	6.20%	-.10

Historic Simulation Sensitivity Approach

Product	Position Size	Sensitivity
UST 10 Year Note 5.5%	\$10MM	\$7,600/bp

Sensitivity-Based P&L

<u>Date</u>	<u>Price</u>	<u>Price</u>	<u>Rate</u>	<u>Change</u>
Jan2	100	99	6.25%	.12
Jan3	101	97	6.35%	.10
Jan4	102	99	6.40%	.05
Jan5	101	98	6.15%	-.25
.
.
.
Dec31	100	94	6.20%	-.13

$P\&L = -\$7,600 * 10bps = -\$76,000$

Estimate P&L for Each Day

<u>Date</u>	<u>Price</u>	<u>Price</u>	<u>Rate</u>	<u>Change</u>		<u>P&L</u>
Jan2	100	99	6.25%	.12>	-91,200
Jan3	101	97	6.35%	.10>	-76,000
Jan4	102	99	6.40%	.05>	-38,000
Jan5	101	98	6.15%	-.25>	190,000
.
.
.
Dec31	100	94	6.20%	-.13>	98,800

Rank Order P&L Results

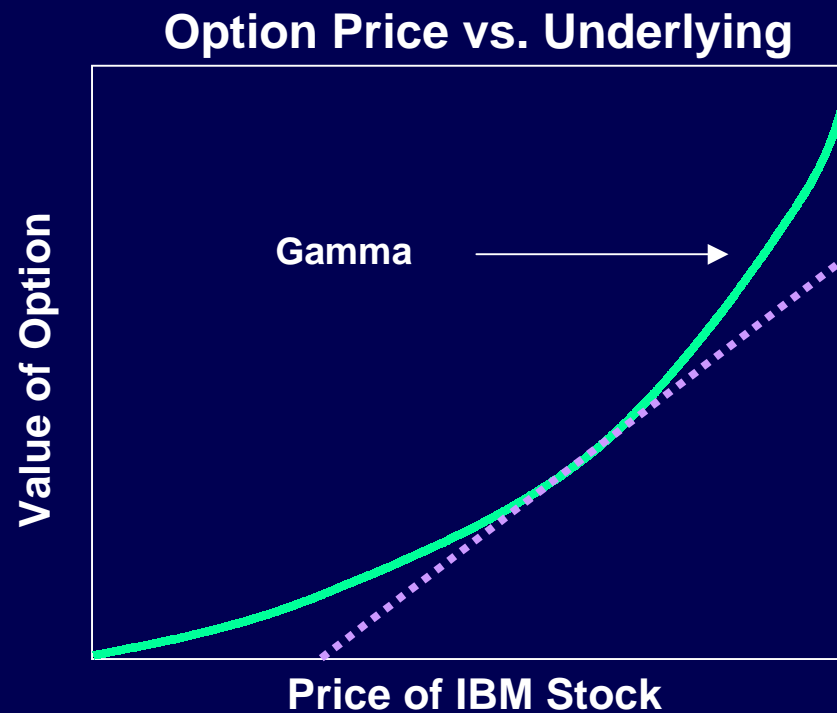
<u>Date</u>	<u>Price</u>	<u>Price</u>	<u>Rate</u>	<u>Change</u>		<u>P&L</u>	<u>Ordered P&L</u>
Jan2	100	99	6.25%	.10	→	-91,200	+190,000
Jan3	101	97	6.35%	.12	→	-76,000	+98,800
Jan4	102	99	6.40%	.05	→	-38,000	-38,000
Jan5	101	98	6.15%	-.25	→	190,000	.
.
.
.	-76,000
Dec31	100	94	6.20%	-.13	→	98,800	-91,200

x% Confidence VAR

<u>Date</u>	<u>Rank</u>	<u>P&L</u>	
Jan 13	1	+229,000	
Mar 4	2	+217,500	
Jun 30	3	+215,300	
Oct 20	4	+199,100	
...	
Jul 7	950	-125,000	← 95% VAR
...	
Mar 12	990	-180,000	← 99% VAR
...	
Apr 19	998	-221,000	
Apr 2	999	-239,000	
Dec 21	1,000	-308,400	

Gamma VAR

Unlike the Variance Covariance approach, the Historic Simulation Sensitivity approach can produce VAR results that fully account for option convexity.



Sensitivity P&L

$$\text{VAR} \equiv \text{delta} (P_t - P_{t-1})$$

Sensitivity to a Unit
Change in Price

Historic Change
in Price

Sensitivity P&L with Gamma Adjustment

$$\text{VAR} \equiv \text{delta} (P_t - P_{t-1}) + \frac{1}{2} \text{gamma}_i (P_t - P_{t-1})^2$$

The first term multiplies the position by the change in price/rate. This calculates the P&L of the positions for the delta risk only. For assets that are not interest rate sensitive and are not options, this is the total P&L.

The second term then adds or subtracts an amount to adjust for the convexity or gamma effect. If the portfolio is net long options and interest rate sensitive assets, the second term reduces loss/increases the gain and thus the risk. If the portfolio is net short interest rate sensitive positions and options, the second term increases the loss/reduces the gain and risk goes up.



P&L Example

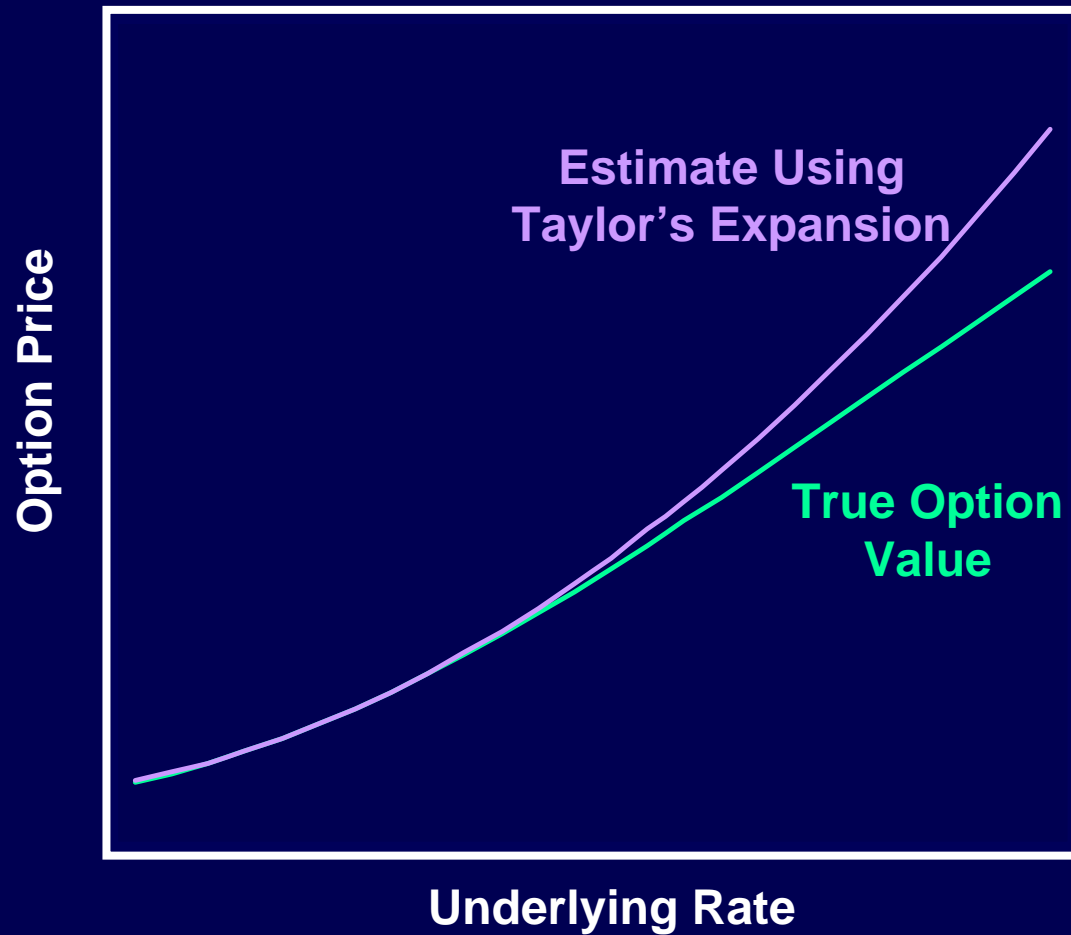
Using Taylor's Expansion

Sample Position

Sensitivity	Position Size	Sensitivity	Daily Volatility	1 Day VAR
Delta Risk (Duration)	\$10MM	\$7,600/bp	10 basis points	\$76,000
Gamma Risk (Convexity)	\$10MM	-\$400 for 10 bps	10 basis points	\$2,000
TOTAL	\$10MM	na	10 basis points	\$78,000

$$-\$7,600 * 10\text{bps} + \frac{1}{2} * -400 / 10 * 10^2 = -\$78,000$$

Taylor's Estimation



Estimation Error One Gamma Taylor's

Stock Price	Black & Scholes	Taylor's Estimate	% Difference
85	.06	2.53	?
90	.36	.83	?
95	1.40	1.39	1%
100	3.64	3.64	0%
105	7.13	7.20	1%
110	11.51	12.07	5%
115	16.30	18.26	12%
120	21.25	25.77	22%

Multiple Gamma Inputs

The accuracy of the Taylor's estimate can be improved by applying gamma sensitivities for multiple shifts in the underlying.

Change in Underlying Rate

Value of 01

-75	-25	-10	0	+10	+25	+75
\$4,000	\$5,800	\$6,600	\$7,000	\$7,300	\$7,500	\$7,600

Gamma Sensitivities

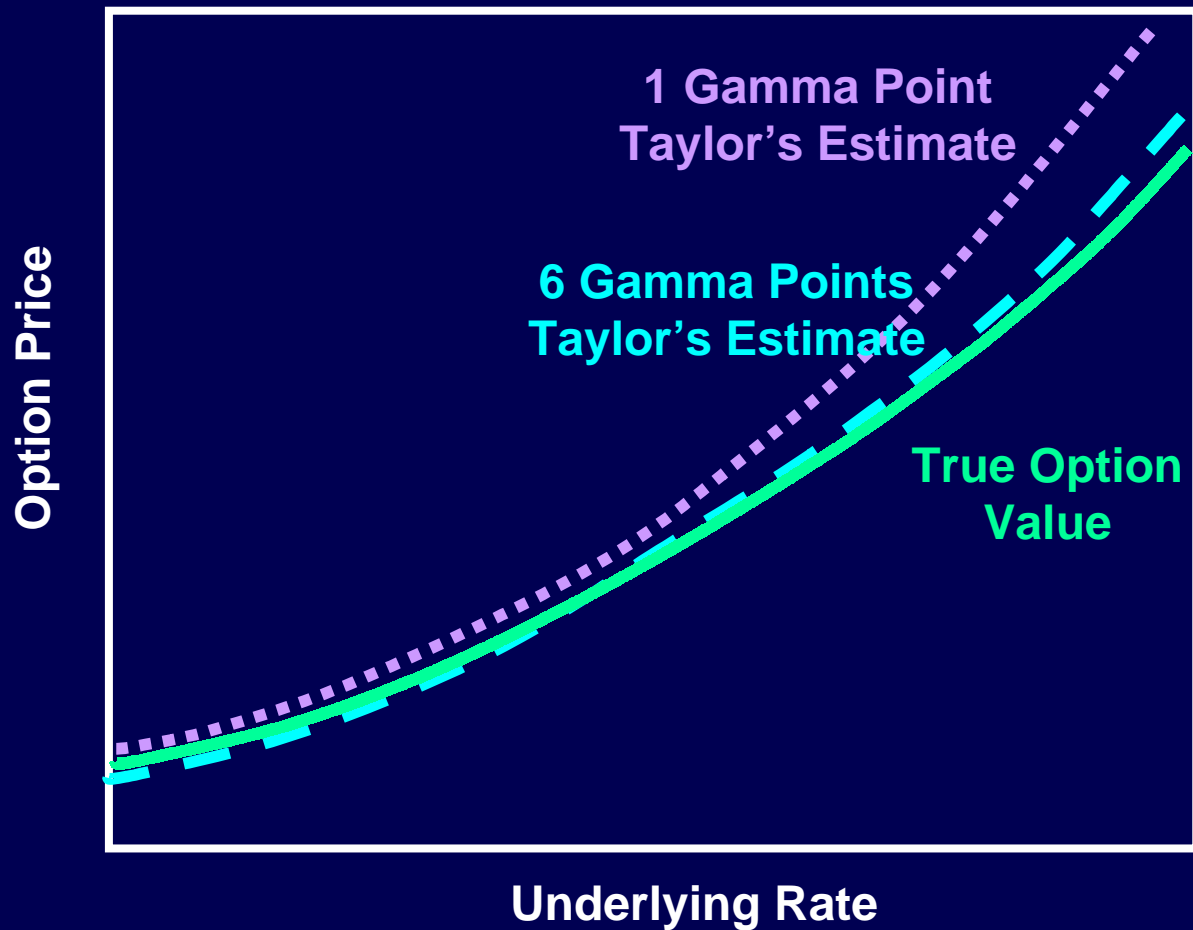
$$\text{delta } (r_t - r_{t-1}) + \frac{1}{2} \text{gamma}_i (r_t - r_{t-1})^2 = \text{P \& L}$$

$$7,000 * -10\text{bps} + 1/2 * -400 / 10 * 10^2 = -\$72,000$$

$$7,000 * -25\text{bps} + 1/2 * -1,200 / 25 * 25^2 = -\$187,500$$

$$7,000 * -75\text{bps} + 1/2 * -3,000 / 75 * 75^2 = -\$637,500$$

Taylor's Estimate Using Multiple Gamma Taylor's



Estimation Error

Multiple Gamma Taylor's

Stock Price	Black & Scholes	Taylor's Estimate	% Difference
85	.06	0	na
90	.36	0	na
95	1.40	1.43	0%
100	3.64	3.64	0%
105	7.13	7.19	1%
110	11.51	11.65	1%
115	16.30	16.45	1%
120	21.25	22.40	5%

Why Doesn't Everyone Use Multiple Gammas?

- ◆ Many front office systems cannot provide gamma sensitivities for multiple shifts in prices/rates
- ◆ Calculating gamma can be time consuming
- ◆ Data management may not be capable of storing multiple gamma sensitivities
- ◆ Still leaves some room for error (at extreme price changes)

Historic Simulation Sensitivity Approach

Advantages

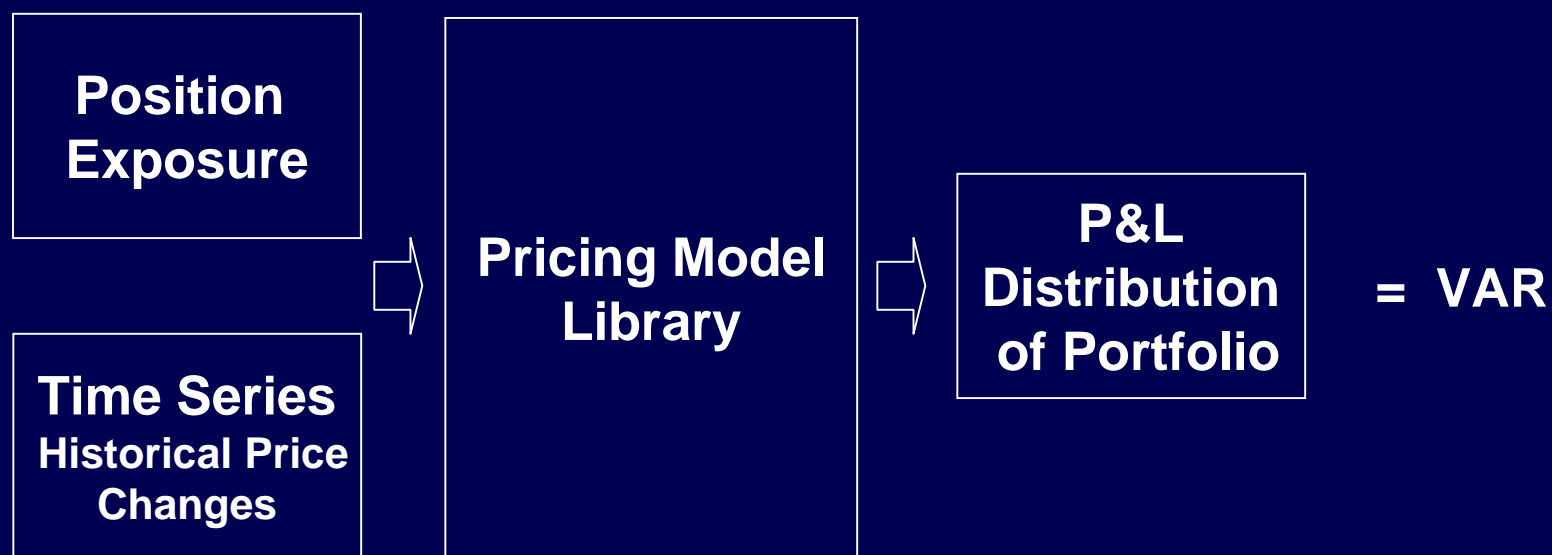
- ◆ Fast since it doesn't revalue every position
- ◆ Works for linear risks and options risk
- ◆ Requires only portfolio level sensitivities

Disadvantages

- ◆ Does not revalue positions
- ◆ Can only estimate complex or discontinuous payoffs

Historic Simulation Full Revaluation

The Historic Simulation Full Revaluation approach calculates P&L by revaluing the portfolio given historic in prices/yields.



Historic Simulation Revaluation P&L

Time Series

<u>Date</u>	<u>Price</u>
Jan2	130
Jan3	131
Jan4	132
Jan5	130.5
.	.
.	.
.	.
Dec31	130

Pricing Models

Fixed Income
Options
MBS/CMO
Other

Daily P&L

<u>Date</u>	<u>P&L</u>
Jan2	-\$1,200
Jan3	+2,581
Jan4	+2,687
Jan5	-3,547
.	.
.	.
.	.
Dec31	+5,415

Rank Order the P&L

<u>Date</u>	<u>Rank</u>	<u>P&L</u>	
Jan 13	1	+229,000	
Mar 4	2	+217,500	
Jun 30	3	+215,300	
Oct 20	4	+199,100	
...	
Jul 7	950	-125,000	← 95% VAR
...	
Mar 12	990	-180,000	← 99% VAR
...	
Apr 19	998	-221,000	
Apr 2	999	-239,000	
Dec 21	1,000	-308,400	

Historic Simulation Full Revaluation

Advantages

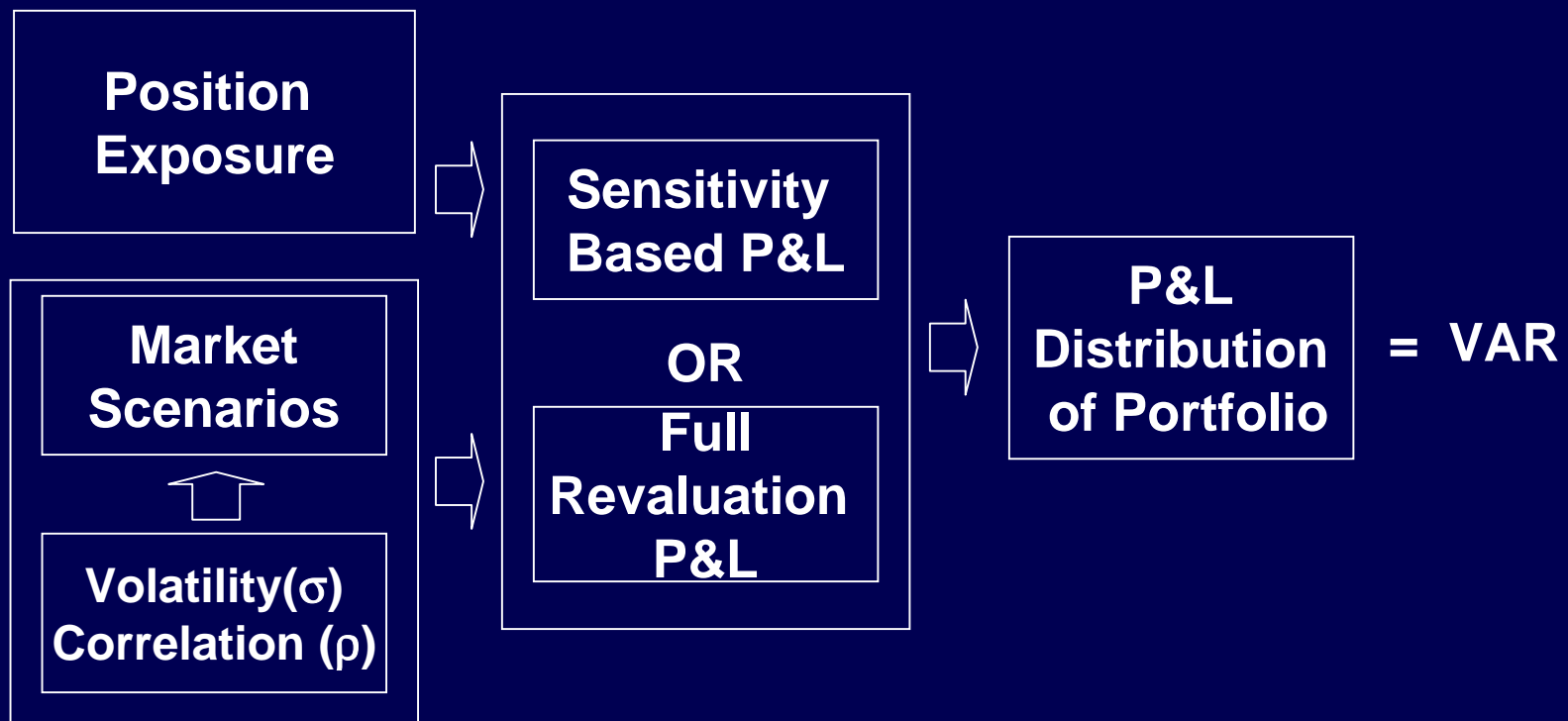
- ◆ Produces accurate P&L results
- ◆ Can model very complex and discontinuous payoffs
- ◆ Works well for linear risks and options risk

Disadvantages

- ◆ Slow and computationally intensive
- ◆ Requires maintenance of a pricing model library
- ◆ Requires trade level detail data inputs

Monte Carlo Simulation

Monte Carlo Simulation generates numerous random market scenarios using predetermined parameters for price volatility and correlation and calculates the P&L for each scenario.



Monte Carlo Simulation

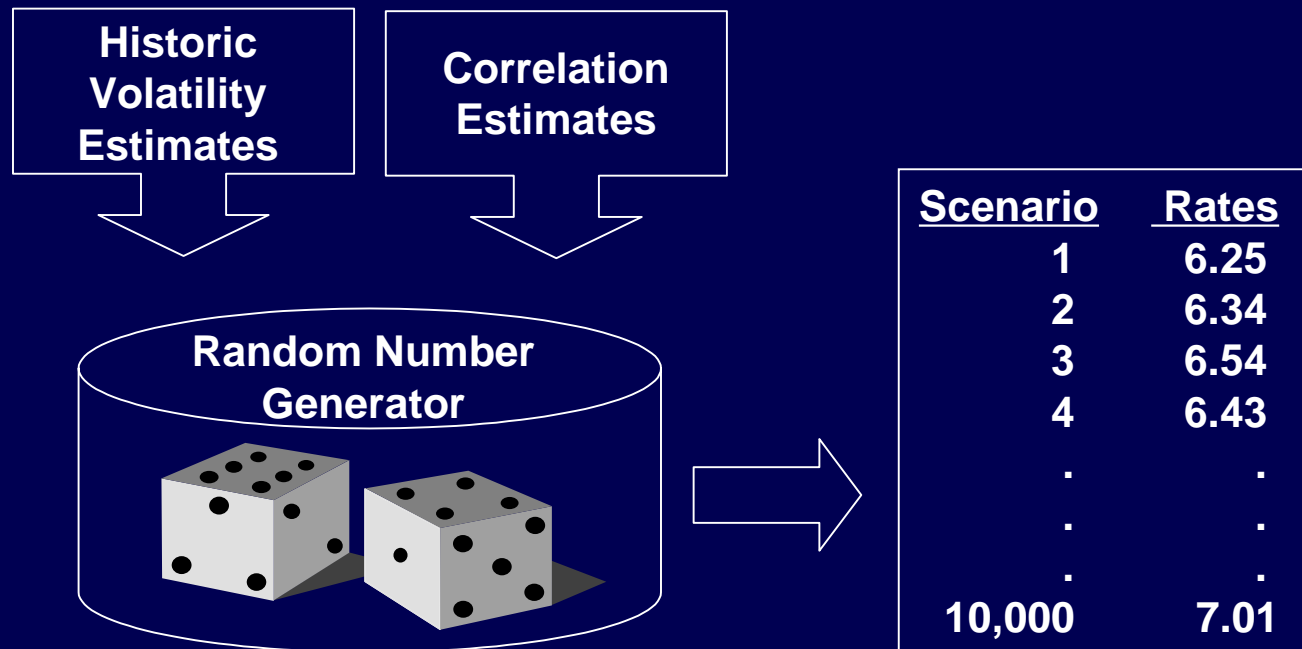
Advantages

- ◆ Produces a distribution of P&L changes
- ◆ Allows for multiple periods with rehedging and maturation
- ◆ Provides greatest level of control over price volatility

Disadvantages

- ◆ Mathematically intensive (scenario generation)
- ◆ Parametric - requires distribution and correlation assumptions
- ◆ Less transparent

Generate Random Scenarios



Calculate the P&L Results for Each Scenario and Rank Order

<u>Scenario</u>	<u>Date</u>	<u>Price</u>	<u>Price</u>	<u>Rate</u>	<u>Change</u>		<u>P&L</u>
1	Jan2	100	99	6.25%	.12▶	-91,200
2	Jan3	101	97	6.35%	.10▶	-76,000
3	Jan4	102	99	6.40%	.05▶	-38,000
4	Jan5	101	98	6.15%	-.25▶	190,000
.
.
.
10,000	Dec31	100	94	6.20%	-.13▶	98,800

Count Down to Find the x% Confidence VAR

<u>Scenario</u>	<u>Rank</u>	<u>P&L</u>	
3,197	1	+229,000	
7,349	2	+217,500	
7,768	3	+215,300	
8,889	4	+199,100	
...	
8,256	950	-125,000	← 95% VAR
...	
7,596	990	-180,000	← 99% VAR
...	
1,991	998	-221,000	
8,467	999	-239,000	
5,469	1,000	-308,400	

Monte Carlo Sensitivity Approach

Advantages

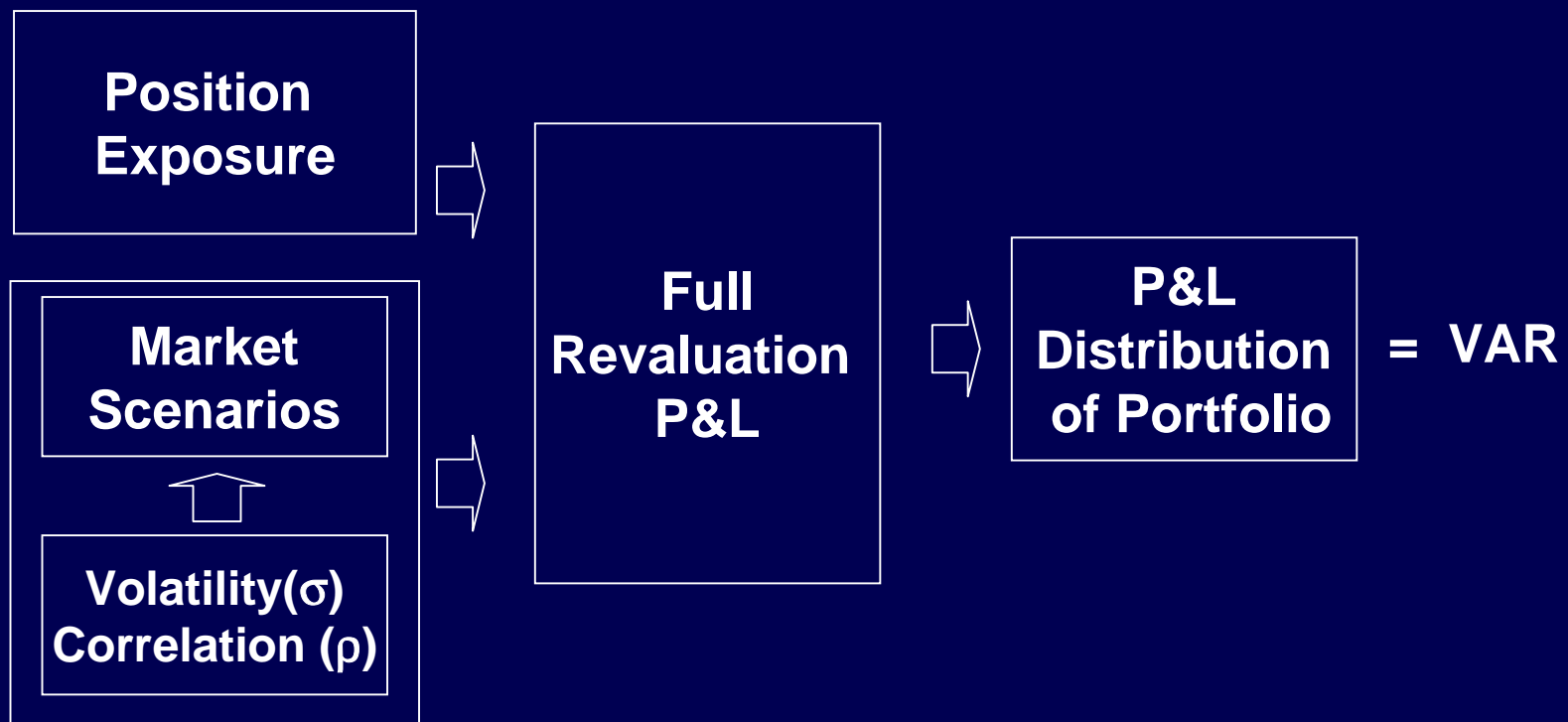
- ◆ Fast since it doesn't revalue every position
- ◆ Works for linear risks and options risk
- ◆ Requires only portfolio level sensitivities

Disadvantages

- ◆ Does not revalue positions
- ◆ Can only estimate complex or discontinuous payoffs

Monte Carlo Full Revaluation

The Historic Simulation Full Revaluation approach calculates P&L by revaluing the portfolio given historic in prices/yields.



Monte Carlo Simulation Revaluation P&L

Scenario Prices

<u>Date</u>	<u>Price</u>
Jan2	130
Jan3	131
Jan4	132
Jan5	130.5
.	.
.	.
.	.
Dec31	130

Pricing Models

Fixed Income
Options
MBS/CMO
Other

Daily P&L

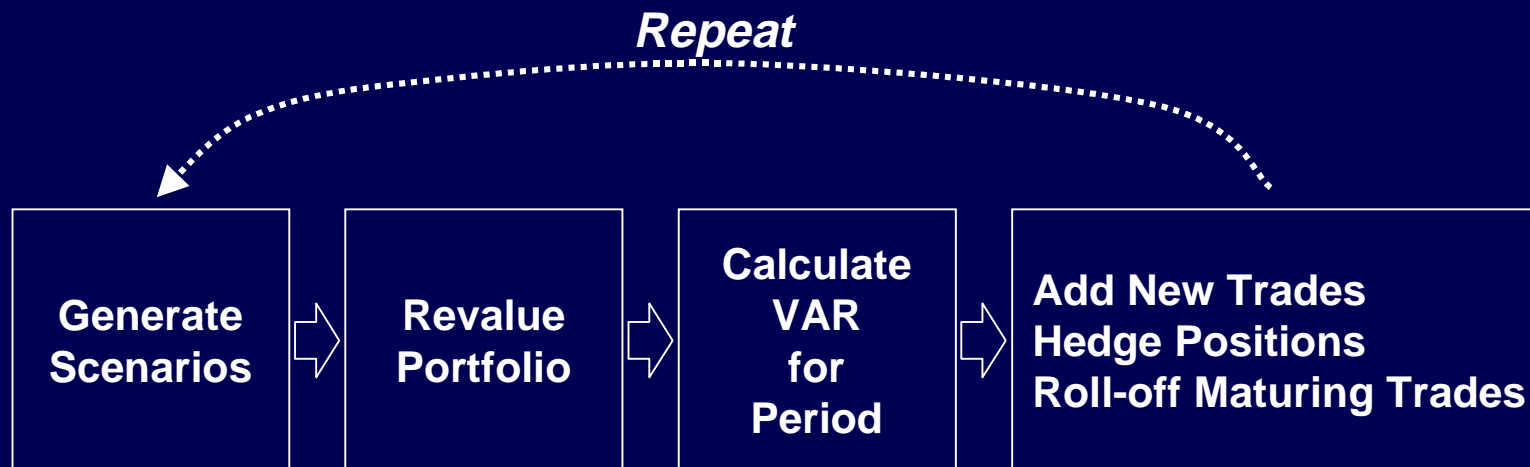
<u>Date</u>	<u>P&L</u>
Jan2	-\$1,200
Jan3	+2,581
Jan4	+2,687
Jan5	-3,547
.	.
.	.
.	.
Dec31	+5,415

Rank Order the P&L

<u>Date</u>	<u>Rank</u>	<u>P&L</u>	
Jan 13	1	+229,000	
Mar 4	2	+217,500	
Jun 30	3	+215,300	
Oct 20	4	+199,100	
...	
Jul 7	950	-125,000	← 95% VAR
...	
Mar 12	990	-180,000	← 99% VAR
...	
Apr 19	998	-221,000	
Apr 2	999	-239,000	
Dec 21	1,000	-308,400	

Multiple Time Horizons

The Monte Carlo Simulation allows for multiple period VAR.



Monte Carlo Full Revaluation

Advantages

- ◆ Produces accurate P&L results
- ◆ Can model very complex and discontinuous payoffs
- ◆ Works well for linear risks and options risk

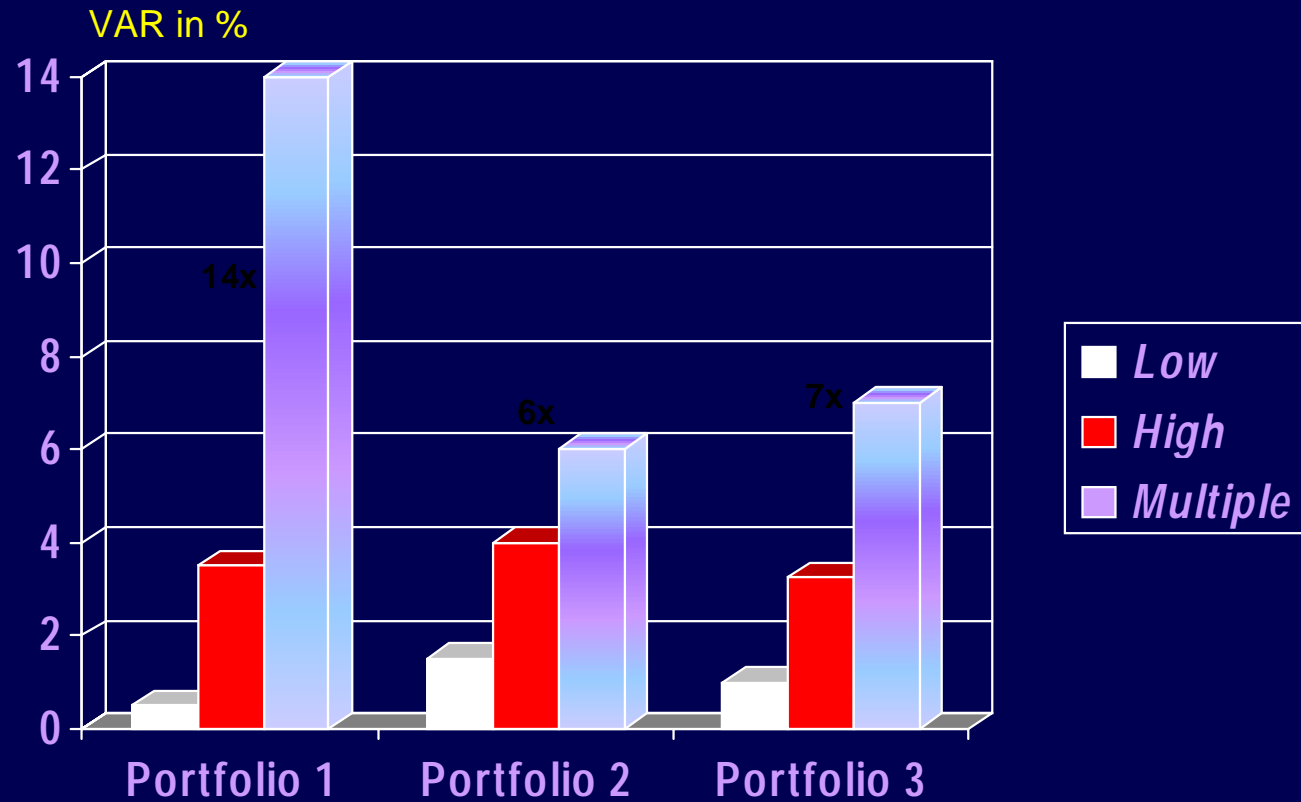
Disadvantages

- ◆ Slow and computationally intensive
- ◆ Requires maintenance of a pricing model library
- ◆ Requires trade level detail data inputs

Three Types of VAR

Method	Advantages	Disadvantages
Variance/ Covariance	<ul style="list-style-type: none"> • Easy to understand • Least computationally intensive • Widely used in industry • Easiest to implement 	<ul style="list-style-type: none"> • Does not fully capture non-linear risks • Assumes normally distributed returns and constant volatilities • Does not capture “Fat Tails”
Historic Simulation	<ul style="list-style-type: none"> • Easy to understand • Can capture non-linear risks • Actual distribution • Incorporates “Fat Tails” 	<ul style="list-style-type: none"> • Can be data intensive • Assumes past is a fair representation of the present
Monte Carlo Simulation	<ul style="list-style-type: none"> • Accommodates a variety of statistical models and assumptions • Can capture non-linear risks, • Can apply multiple time periods 	<ul style="list-style-type: none"> • Does not capture “Fat Tails” • Computationally intensive • Less transparent/more difficult to understand

Highest & Lowest VARs Using Eight Common Techniques



Source: CMRA Survey of Market Practice, 1995

Variance Covariance

Position Inputs

- Delta $\frac{dS}{dR}$
- Gamma $\frac{d^2S}{dR^2}$

Volatility Inputs

- σ of rates or prices
- ρ of rates or prices

Skewness Adjustment

Calculates the z-score adjustment (γ) factor due to the lack of symmetry of an option payoff

$$3 \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^N \text{delta}_i \text{delta}_j \text{gamma}_k \text{cov}_{i,j} \text{cov}_{j,k} + \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^N \text{gamma}_i \text{gamma}_j \text{gamma}_k \text{cov}_{i,j} \text{cov}_{j,k} \text{cov}_{i,k}$$

P&L Calculation

Calculates the P&L by multiplying the position (delta) times the change in price or rate for one standard deviation. Then adjusts for diversification benefits.

$$\sum_{i=1}^N \text{delta}_i^2 \sigma_i^2 + \sum_{i=1}^N \sum_{\substack{j=1 \\ i \neq j}}^N \text{delta}_i \sigma_i \text{delta}_j \sigma_j \rho_{i,j}$$

Z-Score

Confidence # σ 's

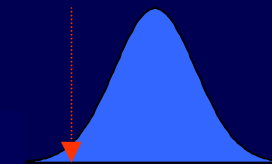
84%	X
90%	X
95%	X
97%	X
99%	X

VAR Result

VAR Loss for one standard deviation
(Normal Distribution)

99% VAR Result

99% VAR = 84% VAR * γ 2.33



Historic Simulation Sensitivity

Position Inputs

- Delta dS
 dR
- Multiple Gammas $d''S$
 dR''

Time Series Inputs

Date	Price	Price	Rate
Jan2	100	99	6.25%
Jan3	101	97	6.35%
Jan4	102	99	6.40%
Jan5	101	98	6.15%
.	.	.	.
.	.	.	.
Dec31	100	94	6.20%

P&L Calculation

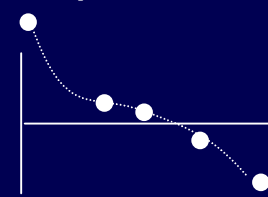
Calculates the P&L by multiplying the position (delta) times the actual change in price or rate for day in history. The second term multiplies the one half gamma position times the change in position squared. This is the addition/subtraction for the "option effect".

$$\sum_{t=1}^T \text{delta} (P_t - P_{t-1}) + \sum_{t=1}^T \frac{1}{2} \text{gamma}_p (P_t - P_{t-1})^2$$

The gamma you apply depends on the size of the price/ rate shock

Gamma Curve

Spline Fit



Position Inputs

Gamma Vector

Shock	Gamma
+100	10
+25	2
0	1
-25	-2
-100	-5

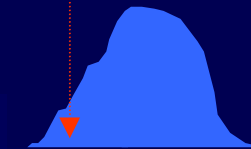
VAR Results

Sorted P&L

Date	P&L
Mar2	250
Jun15	130
Apr4	19
Apr19	-125
.	.
Nov30	-353

99% VAR Result

99% VAR = 99% Percentile Result



Historic Simulation Full Revaluation

Position Inputs

- Size
- Maturity
- Coupon Rate
- Volatility
- Strike
- Yield Curve
- Other

Time Series Inputs

Date	Rate	Rate	Rate
Jan2	7.2%	5.2%	6.2%
Jan3	7.3	5.2	6.2
Jan4	7.4	5.2	6.2
Jan5	7.2	5.2	6.2
⋮	⋮	⋮	⋮
Dec31	8.1%	5.9%	6.20%

P&L Calculation

Calculates P&L by repricing each position using the prices/rates that we actually observed for a day in history. If there are 250 days, each position must be revalued 250 times.

Pricing Model Library

- Options
- Fixed Income
- MBS/CMO
- Other

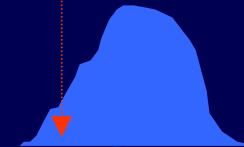
VAR Results

Sorted P&L

Scenario	P&L
6,795	250
1,498	130
948	19
239	-125
⋮	⋮
5,394	-353

99% VAR Result

99% VAR = 99% Percentile Result



Monte Carlo Simulation Sensitivity

Position Inputs

- Delta $\frac{dS}{dR}$
- Multiple "S Gamas dR"

Volatility Inputs

- σ of rates or prices
- ρ of rates or prices

Scenario Generator

Scenario	Price	Price	Rate
1	100	99	6.25%
2	101	97	6.23%
3	102	99	6.65%
4	101	98	6.15%
.	.	.	.
.	.	.	.
9,998	100	94	7.25%
9,999	100	94	7.25%
10,000	100	94	7.25%

P&L Calculation

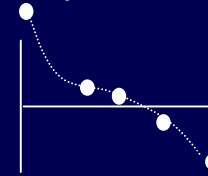
Calculates the P&L by multiplying the position (delta) times the actual change in price or rate for day in history. The second term multiplies the one half gamma position times the change in position squared. This is the addition/subtraction for the "option effect".

$$\sum_{t=1}^T \text{delta}(P_t - P_{t-1}) + \sum_{t=1}^T \frac{1}{2} \text{gamma}_p (P_t - P_{t-1})^2$$

The gamma you apply depends on the size of the price/ rate shock

Gamma Curve

Spline Fit



VAR Results

Sorted P&L

Date	P&L
Mar2	250
Jun15	130
Apr4	19
Apr19	-125
⋮	⋮
Nov30	-353

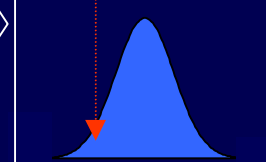
Position Inputs

Gamma Vector

Shock	Gamma
+100	10
+25	2
0	1
-25	-2
-100	-5

99% VAR Result

99% VAR = 99% Percentile Result



Monte Carlo Simulation Full Revaluation

Position Inputs

- Size
- Maturity
- Coupon Rate
- Prepayment speed
- Today's Price
- Today's Rates

Volatility Inputs

- σ of rates or prices
- ρ of rates or prices

Scenario Generator

Scenario	Price	Price	Rate
1	100	99	6.25%
2	101	97	6.23%
3	102	99	6.65%
4	101	98	6.15%
...
9,998	100	94	7.25%
9,999	100	94	7.25%
10,000	100	94	7.25%

P&L Calculation

Calculates P&L by repricing each position using a scenario that was generated based on historic estimates of volatility and correlation. If there are 10,000 scenarios, each position must be revalued 10,000 times.

Pricing Model Library

- Options
- Fixed Income
- MBS/CMO
- Other

VAR Results

Sorted P&L

Scenario	P&L
6,795	250
1,498	130
948	19
239	-125
...	...
5,394	-353

99% VAR Result

99% VAR = 99% Percentile Result

