Portfolio Management: Portfolio Management Applications Using Derivatives
CFA Level III - Study Session 13

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Reading Assignments

1. Futures, Options & Swaps, 2nd edition, Robert W. Kolb (Blackwell, 1997)

   A. "Interest Rate Futures: Refinements," Ch. 6, pp. 189–217
   B. "Stock Index Futures: Refinements," Ch. 8, pp. 255–265
   C. "Option Payoffs and Option Strategies," Ch. 11, pp. 373–383
   D. "Option Sensitivities and Option Hedging," Ch. 14, pp. 461–471
   E. "The Swaps Market: Refinements," Ch. 20 (revised), pp. 650–674

2. "Minimizing Cash Drag with S&P 500 Index Tools," Joanne M. Hill and Rebecca K. Cheong, Equity Derivatives Research (Goldman, Sachs, June 11, 1996, revised)

3. "Using Interest Rate Futures in Portfolio Management," Concepts & Applications (Board of Trade of the City of Chicago, 1988) (Candidates should refer to Kolb Chs. 5–8 and 14.)

Key Points

1) Understand portfolio applications of the derivatives

2) Derivatives provide alternative means of reducing or increasing exposure to interest rate and equity market moves

3) Understand immediate sensitivity of derivatives to the underlying variables (hedge ratios)

4) Understand the transaction cost and tax implications of using derivatives

5) Understand advantages and disadvantages of OTC and exchange traded derivatives

Learning Outcomes

1. A. **"Interest Rate Futures: Refinements"**

a) **Construct** a strategy using interest rate futures that will synthetically shorten or lengthen the duration of a U.S. Treasury bill investment, including computing the number of T-bill futures contracts required;

b) **Create** a synthetic floating-rate (fixed-rate) loan from a fixed-rate (floating-rate) loan including computing the number of Eurodollar futures required;

c) **Compare and contrast** "strip" and "stack" hedges, including **computing** the number of futures contracts required;

d) **Compare and contrast** the various hedging strategies or models (MVN, CF, BP, RGR, and the PS) that can be used when hedging with T-bond futures, including computations using the various methodologies;

e) **Compare and contrast** the differences between a cross-hedge and a perfect hedge, including computing the number of futures contracts and an ex post evaluation showing the distinctions between the two strategies;

f) **Construct, formulate, and evaluate** an immunization strategy using interest rate futures (refer to Study Session 6, Reading 1-C).

1.A.a. Shorten and Lengthen Duration of T-Bill Portfolios

- T-bills are discount instruments
  Spot or Discount Price = DP = FV – [(DY * FV *DTM)/360]

- T-Bill Contracts: Underlying is a T-bill that matures 90 days after the contract expires
  T-bill contract notional size is 1,000,000 par (future value)

- Shorten maturity: i.e. Go from Tau to Tau-90
  Sell future expiring in Tau-90
  Sell no. of contracts = [Face Value of Your T-Bills / Notional Size Per Contract]

- Lengthen maturity - Go from Tau to Tau+90
  Buy future expiring in Tau
  Buy no. of contracts = [Face Value of Your T-Bills / Notional Size Per Contract * Spot Price/100]

1.A.b. Convert Floating (Fixed) Rate Loans To Fixed (Floating) Rate Loans

- Eurodollar Contract
  Notional Amount = $1,000,000,
  Price = 100 * [1-$_{exp}FR_{exp+90}$ * 91.25/365]
  BPV=$25 = 1,000,000 * [0.0001 * 0.25]
  Long pays $25 for every basis point rates go up
  Short pays $25 for every basis point rates go down
- Convert Floating Loan to Fixed: Short no of contracts = Loan Face Value / Notional Size
- Convert Fixed Loan to Floating: Long no of contracts = Loan Face Value / Notional Size
- For investors hedging bond positions it is the opposite of the issuers

1.A.c. Strip and Stack Hedges

- Strip Hedge: Uses an equal number of contracts for each futures expiration over the hedging horizon
  - Benefit: No rollover and curve risk
  - Disadvantage: Distant contracts may be illiquid

- Stack Hedge: Uses a single contract liquid contract for all futures expiration over the hedging horizon and the hedge is rolled over on or before the contract is expired

  - Reasons for Stack Hedge: Liquidity
  - Risk of Stack Hedge: Rollover, and nonparallel curve move (Imperfect cross hedge)

1.A.d. Various Hedging Strategies

- Face Value Naïve Model (FVN)
  - Match face value of cash instrument with face value of instruments underlying
  - Problems: Imperfect hedge because of coupon, duration and market value mismatches

- Market Value Naïve Model (MVN)
  - Match market value of cash instrument with face value of instruments underlying
  - Problems: Imperfect hedge because of interest rate sensitivity mismatch

- Conversion Factor Model (CF)
  - HR = - [Cash Market Principal/Futures Market Principal] * Conversion Factor
  - Uses the conversion factor of the cheapest-to-deliver bond
  - Same as FVN except it adjusts the hedge for characteristics of the CTD
  - Problems: Imperfect hedge because of interest rate sensitivity mismatch

- Basis Point Model (BP)
  - HR = - [$BPV_{\text{cash instrument}}/BPV_{\text{futures}}$] * $\text{Beta}_{\text{cash yield, futures yield}}$
  - HR = - [$BPV_{\text{cash instrument}}/BPV_{\text{futures}}$], if both instruments are driven by the same yield
  - Problem: a good model but it ignores convexity differences
- Regression Model (RGR)
  - HR = -[Cov cash value, future value/Variance future value]=-Beta cash value, futures value

- Parameters estimated using a regression model
- Empirically have worked the best

- Price Sensitivity Model (PS)
  - HR for a single unit of cash asset =

$$-[(P_{cash} * ModDur_{cash})/(P_{fut} * ModDur_{fut})]*Beta_{cash\ yield,futures\ yield}$$

1.A.d. Difference Between A Cross Hedge and A Perfect Hedge

- Perfect Hedge: Uses a derivatives instrument with the cash asset as underlying
  - Eliminates uncertainty of future value

- Cross Hedge: Uses a derivatives instrument with a correlated underlying asset with the cash asset
  - Minimizes volatility of future value but could lead to hedging errors due to imperfect correlations

1.A.e. Immunization Strategy Using Futures

- Bring your portfolio duration the same as the liabilities
- Hedge the portfolio duration to zero when there is no liability (perfect immunization)

1. B. "Stock Index Futures: Refinements"

Candidates should be aware that the 'multiplier' of stock index futures can change and that candidates are not expected to memorize current contract specifications for the exam. The appropriate contract specifications will be given on the exam.

a) **Compute** the correct number of index futures contracts required to partially or completely hedge an equity portfolio;

b) **Construct** a strategy that would decrease/increase the beta of a portfolio, including calculating the number of futures contracts;

c) **Create** a synthetic T-bill or a synthetic equity position using the appropriate futures contracts;

d) **Construct** a portfolio insurance strategy, including outlining the steps in implementing the strategy via dynamic hedging.

1.B.a. Partially or Completely Hedge an Equity Portfolio

- Unadjusted Hedge = - Portfolio Value / (Futures Price * Multiplier)
- Multiplier = Value of One Point
- Adjusted Hedge = Unadjusted Hedge * $\text{Beta}_{\text{portfolio, futures}}$
- Do this with the portion of the portfolio you wish to hedge to cash

1.B.b. Increase or Decrease Beta of An Equity Portfolio

- Figure out the total number of contracts for risk-minimizing hedge (TRMH)
- To double beta of portfolio one would go long the same number of TRMH contracts
- To half your beta short half as many contracts as TRMH
- Final Beta = Initial Beta * [1+No. Contracts/ TRMH]
- Partial Hedge = [TRMH] *[$\text{Beta}_{\text{portfolio, futures}}$-Target Beta]

1.B.c. Create a Synthetic T-bill or Synthetic Equity Portfolio

- Synthetic T-bill: Hold stocks and short TNRMH contracts
- Synthetic Stock = Hold cash and go long TNRMH contracts

1.B.d. Portfolio Insurance Strategy

- To protect portfolio value to fall below a certain value
- Portfolio Insurance = 1 Share + 1 Put = 1 T-bill + 1 Call
- Portfolio Insurance = 1 Share – [HR * (1-Delta)] futures
- HR = No. of contracts to turn shares into cash
- Chose a floor value, Evaluate Volatility, Set the Initial Hedge Position, Dynamically Rebalance
- Delta goes up as prices rises and decreases as prices go down
- Buy in rising markets and sell into falling markets

1.C. "Option Payoffs and Option Strategies"

a) **Construct** a portfolio hedging (insurance) strategy using put options;

b) **Compare** and contrast a hedging strategy that uses put options with a strategy that uses index futures (see Kolb, pp. 263–265);

c) **Evaluate** how a portfolio hedging (insurance) strategy that uses put options affects the distribution of terminal portfolio value;

d) **Draw and evaluate** the profit and loss diagram of the portfolio hedging strategy;

e) **Compare** a strategy in which a call is purchased (sold) and a put is sold (purchased) with a strategy of simply purchasing (selling) the underlying asset;

f) **Determine** whether an arbitrage opportunity exists, given a put price, a call price, the stock price, and the price of a bond;

g) **Construct and evaluate** the appropriate trade when an arbitrage opportunity exists given a put price, a call price, the underlying asset price, and the price of a bond;

h) **Create** a synthetic security from three of the following four instruments: a call, a put, the underlying asset, and a bond.

1.C.a. Construct a Portfolio Hedging (Insurance) Strategy Using Put Options

- Protective Put Strategy = Asset Portfolio + Put Option – Put Premium
- To protect portfolio value to fall below a certain value
- Upside = Underlying Upside – Put Premium

1.C.b. Contrast Hedging Strategy Using Put options and Index Futures

- Protective Put Strategy = Asymmetric Horizon Return Profile
- Future Hedge = Symmetric Horizon Return Profile
- To create asymmetric profile using future need dynamic hedging
- Puts are not marked to market daily whereas futures are

1.C.c. The Distribution of Terminal Portfolio Value

- Protective Put Strategy insures a floor for the portfolio value

1.C.d. P&L Diagram for Portfolio Hedging Strategy

1.C.e. Compare Strategy of Buying (Selling) a Call and Selling (Buying) a Put with Buying (Selling) and Asset

- Call(S,X, t, Tau) – Put(S,X, t, Tau) = Stock(t) – PV(X, t, Tau)
- Long Call and Short a Put has the same effect of buying the stocks and borrowing the NPV of the strike price
- Short Call plus Long Put has the same effect of selling the stock and lending the NPV of X
- Synthetic Long Stock = Buy Call, Sell Put, and Lend NPV(X)
- Synthetic Stock Short = Sell Call, Buy a Put, and Borrow NPV(X)

1.C.f. Using Put-Call Parity to Determine an Arbitrage Opportunity
1.C.g. Construct and Evaluate the Appropriate Trade When an Arbitrage Opportunity Exists

- Compare Put – Call with Stock – NPV(X)
- If Call – Put > Stock – PV(X) ➔ Sell call, Buy Put, Long Stock, Borrow NPV(X)
- If Call – Put < Stock – PV(X) ➔ Buy call, Sell Put, Sell Stock, Lend NPV(X)

1.D. "Option Sensitivities and Option Hedging"

a) **Analyze** the properties of a delta neutral portfolio and relate those properties to dynamic hedging (see Perold and Sharpe in Study Session 11);

b) **Modify** the delta neutral portfolio and show how it is related to creating a synthetic call using bonds and the underlying asset;

c) **Illustrate** how the delta neutral portfolio is related to portfolio hedging strategies.

Option Sensitivities:

- Delta = [PV1-P0]/[S0-S1] = Portfolio Price Sensitivity to Underlying Asset Price
  - Call Delta increases with spot price
  - Put Delta is negative and its absolute value decreases with spot price
  - Call Delta = 1+ Put Delta

- Theta = Time Decay = [C1-C0]/[t1-t0] all things equal
  - Option time decay increases with getting closer to expiration
  - Time decay is larger for at-the-money options

- Vega = Portfolio Value Sensitivity To Volatility = [PV1-PV0]/[Vol1-Vol0] all things equal except volatility
  - Option vega increases with increasing expiration date

1.D.a. Analyze the Properties of a Delta Neutral Portfolio

- DNP is a portfolio with a zero Delta
- With small changes in stock prices, portfolio market value stays unchanged
- Delta neutral portfolios are immunized for small changes in market moves

1.D.b. Creating Synthetic Call

- DNP = - Call + N1 * S ➔ DNP is same as a position in T-bill
- N1 is Delta of the Call
- Hold Delta Shares of Stock and short T-bill == Replicate Call Option
- Call = Delta Shares – N2 *T-bills

1.D.c. Relation Between the Delta Neutral Portfolio and Dynamic Hedging

- Portfolio Insurance = Stocks and T-bills in a dynamically rebalanced portfolio
- DNP = + Put – N2 * S ➔ DNP is same as len T-bill
- N1 is Delta of the Call
- Sell Delta Shares of Stock and Long T-bill == Replicate Put Option
- Put = - Delta Shares + N1 * T-bills

1.E. "The Swaps Market: Refinements"

a) **Demonstrate** how a swap agreement can be viewed as a pair of bond transactions;

b) **Demonstrate** how a swap agreement can be viewed as a portfolio of forward rate agreements;

c) **Demonstrate** how an interest rate swap agreement can be viewed as a strip of Eurodollar futures contracts;

d) **Demonstrate** how an interest rate swap agreement can be viewed as a portfolio of interest rate options (caps and floors);

e) **Construct** synthetic fixed (floating) rate debt using an interest rate swap and discuss how the synthetic fixed (floating) rate debt can reduce interest rate risk

f) **Construct** and evaluate synthetic callable (noncallable) debt from noncallable (callable) debt using swaptions and discuss how this can reduce interest rate risk;

g) **Construct** synthetic dual currency debt from a U.S. dollar bond using a fixed-for-fixed (circus) swap, and provide appropriate illustration.

1.E.a. Swap as A Pair of Bonds

- Fixed to Floating = Long Floating and Short Fixed
- Currency Swap = Long US Bond Short German Bund

1.E.b. Swap as A Portfolio of FRA

- FRA = Single period fixed-for-floating swap
- Fixed for Floating Swap Transaction = Portfolio of single period fixed-for-floating swaps
  = Portfolio of FRAs

1.E.c. Swap as A Strip of Euros

- Eurodollar Contract = Single period fixed-for-floating swap that is marked to market
- Fixed for Floating Swap Transaction = Portfolio of single period fixed-for-floating swaps
  = Portfolio of Eurodollar contracts = Eurodollar Strip

1.E.d. Swap as A Portfolio of Caps and Floors

- FRA = Long a call (cap) and short a put (floor) on rate (paired collar)
- FRA = Single period fixed-for-floating swap
- Fixed for Floating Swap Transaction = Portfolio of single period fixed-for-floating swaps
  = Portfolio of FRAs
  = Portfolio of pairs of long caps and short floors

1.E.e. Synthetic Fixed or Floater Bond Using Swap and Interest Rate Sensitivity

- Own floating bond, pay floating, and receive fixed on a swap ➔ owning a fixed bond
    - Borrow fixed, pay floating, and receive fix reduces interest rate sensitivity
- Own fixed bond, pay fixed, and receive floating on a swap ➔ owning a floating bond
    - Own fixed, pay fixed, and receive floating reduces interest rate sensitivity

1.E.f. Synthetic Non-callable Debt

- Have issued a callable fixed-rate bond = Non-callable Debt + A Call Option To Receive Fixed
- Sell receiver swaption (option to enter into a swap and pay fixed rate)
- Combination of the above ➔ Fixed Rate Non-callable Debt

1.E.g. Synthetic Dual Currency Debt

- Dual Currency Debt = Borrow in One Currency and Pay in Interest in Another
- Currency Swap = Exchange Principal at the beginning and at the end
- Example:
    - Issue $100mm in dollars and pay 6.5% in USD for 5 years
    - We wish to pay coupon in Euro
    - Enter in a currency swap to pay Euro cash flows and receive dollar cash flows
    - Undo initial and terminal exchange commitments by contracting in the forward markets

2. "Minimizing Cash Drag with S&P 500 Index Tools"

a) **Compare** and contrast the two primary tools used in cash management of equity index portfolios;

b) **Formulate** and evaluate a cash management strategy using futures;

c) **Calculate** and appraise returns between a cash management strategy using index futures or SPDRs with that of leaving a portion of the portfolio in cash;

d) **Formulate** the cost of excess cash versus an appropriate cash management strategy using futures or SPDRs;

e) **Illustrate** similar cash management strategies using alternative index instruments.

2.A. Two Primary Tools in Management of Equity Index Portfolios
2.B . Cash Management of Equity Index Portfolios Using Futures
2.C . Comparison of Returns
2.D. Cost of Excess Cash vs. Two Strategies
2.E. Alternate Index Instruments

- **S&P Index Futures**:
  - Can use appropriate index futures to create a synthetic equity position or conversely create a synthetic T-Bill
  - S&P futures track the S&P, tied by the arbitrage relationships
  - Use of derivatives must be allowed
  - Highly liquid
  - Low transaction cost
  - Can keep a buffer in cash and get the exposure through futures

- **S&P 500 Deposit Receipts (SPDRS, pronounced "spiders")**
  - SPDRS are share in investment trusts that owns the index's constituent securities
  - SPDRS are considered a stock and could be used in portfolios that don't allow derivatives
  - Can use cash to buy SPDR shares to create equity exposure or conversely sell them into cash to free up cash for redemption
  - Can only sell them if portfolio is already long
  - Available in smaller denominations
  - May be less liquid and involve higher transaction costs
  - Does not require permission for use of derivatives

- **Performance Vs Index**:
  - Cash: Weight * (Return on cash – S&P return)
  - S&P Futures: Weight * (Tracking Error – Transaction Costs)
  - SPDRS: Weight * (Tracking Error – Transaction Costs)
- **Essentially the same arguments for other indices but must consider liquidity**

3. "Using Interest Rate Futures in Portfolio Management"

a) **Construct, formulate, and evaluate** a duration (or BPV) increasing or decreasing strategy using bond futures;

b) **Evaluate** the advantages and disadvantages of using financial futures for asset allocation purposes;

c) **Construct, formulate, and evaluate** an asset allocation strategy using stock index futures and bond futures;

d) **Construct, formulate, and evaluate** a portfolio hedging (insurance) strategy using the delta of the appropriate put option and bond futures;

e) **Judge and support** portfolio hedging (insurance) as a dynamic asset allocation strategy using the delta of the appropriate put option and bond futures;

f) **Create** a synthetic instrument using bond futures.

3.A. Duration or BPV Decreasing Strategy Using Bond Futures

- **BPV = (Modified Duration) * Portfolio Market Value * 0.0001**
  - Can calculate at security level and aggregate
  - Do it at portfolio level
  - Modified Duration = Macaulay Duration / (1+Yield/f) where f = frequency

- **No. of Futures Contract = [Target BPV – Current BPV] / [Futures BPV]**

- **Returns are different with or without duration overlay strategy**

3.B. Advantages and Disadvantages of Asset Allocation Using Stock Index and Bond Futures

- **Advantages**
  - Reduced transaction costs
  - Less disruption in multiple manager setting
  - More liquidity
  - Speed
- **Disadvantages**
  - Mispriced futures
  - Basis risk
  - Rollover risk

3.C. Asset Allocation Using Stock Index and Bond Futures
3.D. Portfolio Insurance Using Delta of Put Option and Bond Futures
3.E. Synthetic Instruments Using Bond Futures

- **Cash + Bond Futures will behave similar to CTD bond**
  - Synthetic position may be cheaper because mispriced futures
  - There is basis risk
  - Delivery options can be adverse
  - Rollover risk