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# Application of Quantitative Credit Risk Models in Fixed Income Portfolio Management

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# Application of Quantitative Credit Risk Models in Fixed Income Portfolio Management

**Ron D'Vari[1], Ph.D., Kishore Yalamanchili[2], Ph.D., and David Bai[3], Ph.D.**
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Structural models of credit risk that seek a relationship between default probability and equity prices have been in use for some time. The model's output is a measure referred to as distance-to-default (DD) which can be viewed as z-score of the firm's value from its default barrier level. We utilize a barrier option approach to calculate DD for individual firms. Using recent historical defaults as reported by S&P, we calibrate a relationship between DD and expected default probability (EDP). We extend such a framework to determine a fair market price of credit, or as a tool to construct credit portfolios with optimal risk/return characteristics. The default probability can be used as a credit ranking tool that feeds off observable parameters, and more frequently updated market data such as equity prices. In addition, the simplicity and transparency of the approach makes it an ideal candidate to perform risk management and analysis of large portfolios of credit exposures. The computational efficiency of such an approach would allow a large portfolio to be recalculated daily and allow the computation of value-at-risk and stress tests. We establish some evidence that DD/EDP: 1) is a useful complement to traditional credit analysis, 2) has predictive power, 3) can be used for relative valuation and the pair-wise comparison, 4) is useful in optimizing portfolios of credit for return and volatility characteristics, 5) can be used for collateralized debt obligation (CDO) tranche sizing, and 6) can be used for delta hedging of individual tranches of CDOs.

## 1. Introduction

Until the 1960s, corporate credit analysis was viewed as an art rather than a science because analysts lacked a way to adequately quantify absolute level of credit risk. Though it is hard to pinpoint a beginning to the modern history of quantitative credit assessment, the work done by Altman (1968) could be considered as a pioneer. In this approach, a number of firms are identified, of which some defaulted and some did not. The goal of the analysis is then to examine the firms at a time before the defaults occurred and identify which descriptors of the firms might have best helped distinguish the defaulting firms from those that did not default. A second class of models, referred to as reduced form models (Jarrow and Turnbull 1995; Duffie and Singleton 1999), use information from actual credit prices to extract default probabilities. A third class of models—the ones to which this paper belongs to—are structural form models. These models are derived from the work of Black and Scholes (1973) and Merton (1974), who observed that both equity and debt could be viewed as options on the value of a firm's assets. This implies that equity option pricing techniques can be adapted for use in assessing credit.

In recent years, there has been much interest in linking credit risk with equity market data. Many practitioners have used the simple link between credit spreads and stock prices over the years. However, the summer 1998 crisis, which saw a sharp increase in both credit spreads and equity volatility, accentuated the need to treat equity volatility as a crucial component of credit. In light of this, many financial institutions have started to complement the traditional credit analysis and ratings with equity-based models that are now offered by several risk management companies (Moody's KMV, Riskmetrics Group's CreditGrade, etc).

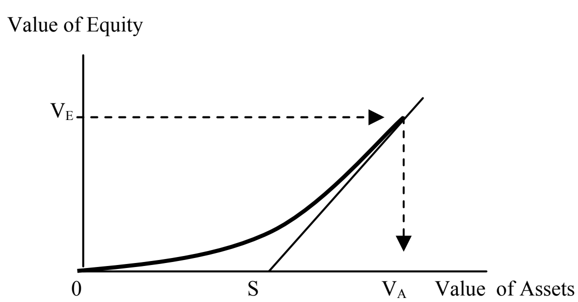
We have also witnessed the exponential growth of credit derivatives in recent years. Initially driven by

[1] Managing Director, Fixed Income Research
[2] Senior Vice President, Associate Director of Quantitative Research
[3] Quantitative Analyst

balance sheet management of bank portfolios, they have since evolved into investment grade and high yield corporate markets as the instruments needed to bridge the gap among various cash securities (bonds, loans, convertible bonds, etc) as well as between market participants (bank portfolio managers, insurance companies, money managers, collateralized debt obligation managers, hedge funds, etc). Sophisticated credit derivatives are now widely used and call for a rigorous quantitative credit risk approach.

This paper is organized in the following way. Section 2 initially presents a barrier options methodology to calculate the firm-level asset value and distance-to-default (DD). Next it provides the empirical evidence for the negative relationships between both distance-to-default (DD) and expected default probability (EDP), and distance-to-default (DD) and spreads. Section 3 describes the procedure— commonly referred to as "factor" model methodology to calculate asset correlation and the (default/non-default) binominal distribution methodology to derive portfolio level credit Value-at-Risk (VaR). Section 4 describes the application for our approach. Concluding remarks are presented in Section 5.

*Figure 1: Equity Value vs. Asset Value*

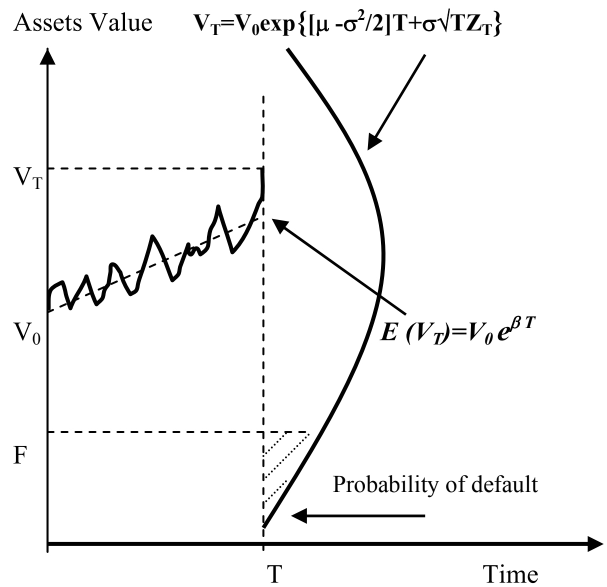



## 2. Single Name Default Risk

An option based model for estimating the default risk of public firms has been developed. This model is based on Merton's structural model (1974) for a firm (Figure 1). The structural model assumes a log normal distribution for a firm's value. Our model uses a barrier option model for valuing a firm with the equity as the call option on the firm asset value. The model uses a combination of short term debt, and one half of the long term debt as the barrier level (Figure 2). The model assumes a one-year time period.

The model's output is distance-to-default (DD). DD is a measure of how many standard deviations the current value of a firm is away from the default level. It can be viewed as z-score of the firm's value from its default level. The lower the DD, the higher the probability of default. The distance to default is not merely a proxy for stock volatility, stock price or any other single variable. Rather it is the result of a model, which includes the non-linear interaction of stock price, volatility, and debt levels.

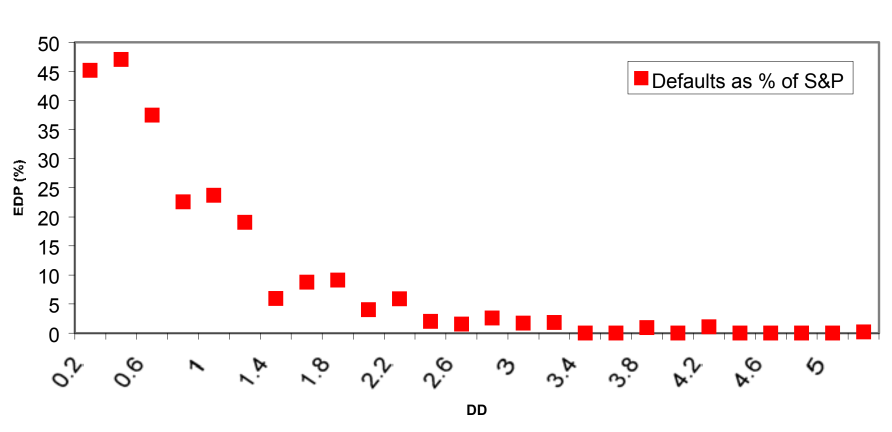
*Figure 2: Barrier Options for Asset Value*




The assumptions in the Merton model result in low default probabilities, which is not consistent with actual default experience. S&P publishes an annual ratings study of the firms they rate. It includes defaults of firms rated by them. We collected this data for 1999, 2000, 2001 and 2002. From the universe of firms in S&P's FactSet, we extracted a subset of public firms that were rated by S&P at the end of 1998, 1999, 2000 and 2001. We calculated distance-to-default for the above subsets at the end of 1998, 1999, 2000 and 2001. A frequency table has been constructed showing the number of S&P rated public firms that have defaulted during the next year (Figure 3).

If there were no relationship between DD and actual default, we would expect to see a random pattern between the two variables. The plots of expected default probability (EDP) vs. DD show that this is not the case. There is clearly a relationship between DD and actual defaults for all the four years considered in this study. The default probability becomes non-zero at a DD of 2.0-2.5 and increases nonlinearly as DD decreases. While defaults have clearly increased from 1999 to 2002, the propensity of low DD firms to default is evident even in 1999. The results clearly

Figure 3: DD and S&P Default Probability Distribution (1999-2002)

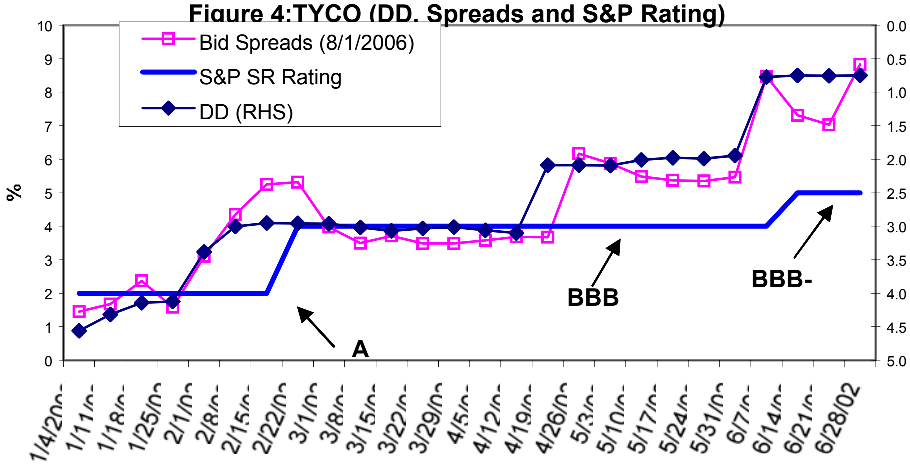


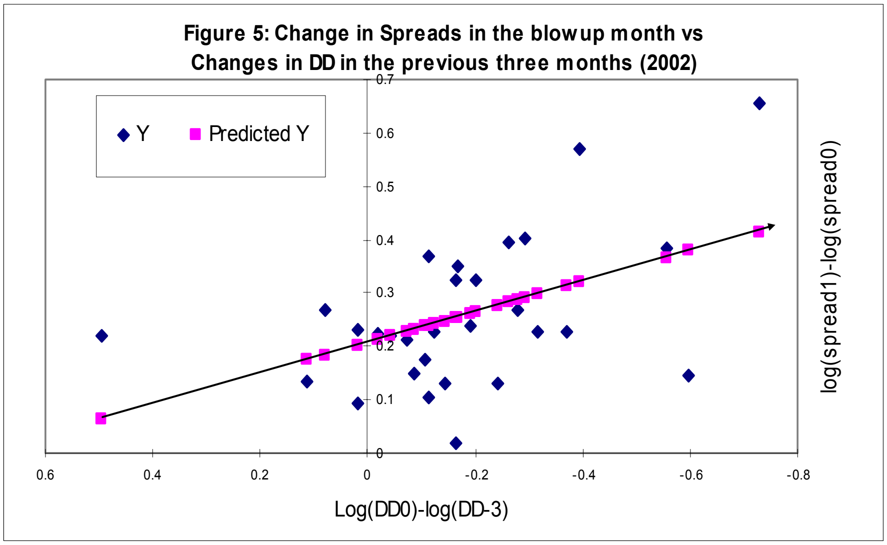

show that a DD of 2.0 or below gives a good indication of future credit risk in a firm.
The DD of a firm is derived from the fundamentals of the firm, and the Merton's structural-form model provides the market's forward-looking views on the credit risk of the firm. Therefore this "quandamental" approach provides predictive power to the firm's bond spread in comparison to the S&P rating (See the example for Tyco International in Figure 4). Rigorous time series analysis provides significant evidence supporting this hypothesis.

We studied the ability of changes in DD to identify corporate bond blowups in advance. Our sample included bonds from 200 largest issuers in Lehman Credit Index having both observable DDs and spreads at the beginning of 2002. Lehman Credit Index includes only bonds of issuers rated investment grade (Baa3 or better) by Moody's Investors Service. Spread blowup is defined as spread widening by 100bp within one month. For a given blowup case, the independent variable is the three-month change in DDs prior to that of blowup. That is, since DDs are purported to be indicative of the likelihood of default with the next year, we chose as a window for the dependent variable in the blowup month following the changes in DD, the previous three months prior to the spread blowup.

Figure 5 is a scatterplot of the changes in DDs (i.e., the independent variable) between $T_{-3}$ and $T_0$ and the resulting percent changes in OAS spreads (i.e. the dependent variable) over the subsequent blowup period for the blowup firms in our study. Larger spreads and smaller DDs indicate decrease in credit quality. The axes in Figure 5 are arranged such that positive values are associated with increases in spreads and decreases in DDs (or increases in expected default probability). Thus, to the extent that decreases in DDs anticipate deterioration in credit quality, subsequent changes in spreads for blowup firms ought to cluster in the upper right-hand quadrant of Figure 5. Of the 28 corporate bond blowups in top-200 Lehman Credit Index names in 2002, 23 names or 82% experienced decreasing DDs

Figure 4:TYCO (DD, Spreads and S&P Rating)






between $T_{-3}$ and $T_0$. Of the 5 blowups not captured in the study, three names have started with DDs smaller than 2. These three names were EETC's of below investment grade airlines secured by aircraft leases. In general, there could be divergence between lessee's credit and the EETC's. If we exclude the three names, we have a hit rate of 92% (23 out of 25 names), that is, smaller DDs predict weaker credit quality 92% of the time in this study. Therefore we conclude that DDs appear to provide an early warning for spread blowups. In addition, DD/EDP can be used for relative valuation and cheap/rich analysis (For example, see Figure 6).

## 3. Portfolio Credit Risk

DD/EDP can be applied to measure credit risk at the portfolio level. First, asset return correlations are derived from a structural model, which links correlations to fundamental factors. By imposing a structure on the return correlations, sampling errors inherent in simple historical correlations are avoided, and a better accuracy in forecasting correlations is achieved. In addition, there is a practical need to reduce dramatically the number of correlations to be calculated. Multi-factor models of asset returns are used to reduce the number of calculated correlation terms to those of the limited number of common factors affecting asset returns.

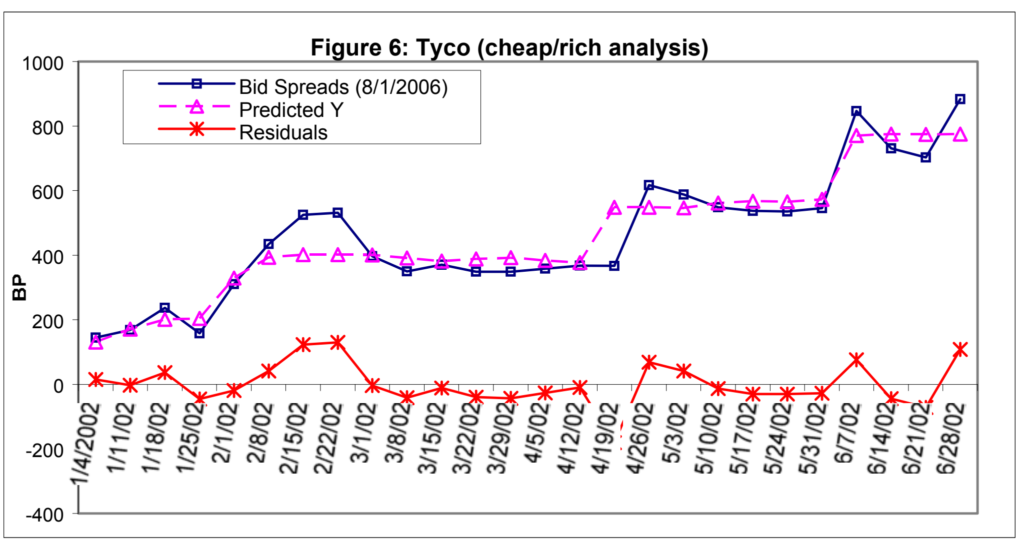
Any firm's return can be decomposed into the following components:

$$[\text{firm return}]=[\text{composite factor return}] + [\text{firm-specific return}] \quad (1)$$

where the composite factor is constructed as the sum of the weighted industry factors:

$$[\text{composite factor return}] = [\text{industry factor returns}] \quad (2)$$

An industry's risk is decomposed into systematic risk arising from sector effect and specific, or



idiosyncratic, risk.

[industry factor returns]= [industrial sector effect]+[industry-specific effect] (3)

Substituting (3) into (2) and plugging (2) into (1), any firm's return can be decomposed into the following components:

[firm return]
=[industrial sector effect]+
[industry-specfic effect]+
[firm-specific effect] (4)

mathematically (4) can be written as

$$r_k=\beta_k\phi_k+\varepsilon_k \quad (5)$$
$$\text{where } \phi_k=\Sigma w_{ki}r_i \quad (6)$$

The factor model for any industry "i" can be expressed by

$$r_i=\Sigma\beta_{iS}\, r_S+\varepsilon_i \quad (7)$$

where $r_S$ = return to factor "S"
$\beta_{iS}$ = industry "i" beta to factor "S"

hence (5) can be rewritten as

$$r_k=\Sigma\beta_{kS}\, r_S+\Sigma\beta_{ki}\,\varepsilon_i+\varepsilon_k \quad (8)$$

where $\beta_{kS}=\beta_k(\Sigma\omega_{ki})\,(\Sigma\beta_{iS})$
$\beta_{ki}=\beta_k(\Sigma\omega_{ki})$
$r_k$=return for firm k
$\phi_k$=composite factors for firm k
$\beta_k$=beta for firm k
$\varepsilon_k$=firm specific effect for firm k

using the factor model we can calculate the correlation matrix. Mathematically this can be written as:

$$\sigma^2(j,k)=\Sigma\beta_{jS}\beta_{kS}\sigma^2_S+\Sigma\beta_{ji}\beta_{ki}\varepsilon^2_I \quad (9)$$

The return correlation between any two firms is related to the covariance in the following way:

$$\rho_{jk}=\sigma(j,k)/(\sigma_j\sigma_k) \quad (10)$$

Next, assuming the defaults/non-defaults binomial distribution, with EDF and asset return correlations, we are able to calculate the portfolio level expected credit loss and its volatility as represented by the 2nd moment of the loss distribution. We use 1.96 times the 2nd moment of the portfolio credit loss distribution as a measure corresponding to credit Value-at-Risk (VaR) at 97.5% confidence level. It should be noted that the determination of 97.5% confidence level assumes that the distribution of portfolio losses are at least approximately normally distributed. This credit VaR measure can be used to quantify absolute level of credit risk and compare the credit risk of the different portfolios. For instance, we can compare the credit risk of the two sample portfolios with and without Tyco International at June 30, 2002 (see Table 1). Portfolio 1 is comprised of 17 equally weighted investment grade issuers. Although Tyco was downgraded in the month June 2002, it was still part of Lehman Credit Index. Portfolio 2 is comprised of the same 17 names plus Tyco and is also equally weighted on all names. Table 1 shows the marginal contribution to expected loss and credit VaR due to this modification. Similarly, one can calculate delta credit VaR, i.e. partial derivative of credit VaR to 1% change in the weight of individual names.

Table 1: Comparison of the credit risk of two portfolios by using DD/EDP

| | Exp Loss | Credit VaR |
|---|---|---|
| Portfolio 1 (w/t Tyco) | 0.57% | 6.10% |
| Portfolio 2 (w. Tyco) | 1.18% | 14.05% |
| Marginal Contribution | 0.61% | 7.95% |

# 4. Application

In practice we find that:

- DD/EDP is a useful complement to traditional credit analysis.
- DD/EDP has predictive power.
- DD/EDP can be used for relative valuation and the pair-wise relative valuation.
- The approach can be used for optimizing portfolios of credit for return and volatility characteristics and marginal contribution to the credit expected loss and credit VaR measures due to adding or reducing exposures in specific names
- DD/EDP can be used for collateralized debt obligation (CDO) tranche sizing, risk assessment, and relative value comparisons.
- DD/EDP can be used for delta hedging specific tranches of CDOs.

# 5. Conclusions

We have presented a simple approach that provides a robust relationship between default probability (or DD), equity price and credit spread. The approach for default probability can be used as credit ranking tool that feeds off observable parameters, and more frequently updated market data such as equity prices. In addition, the simplicity and transparency of the approach makes it an ideal candidate to perform risk management and analysis of large portfolios of credit exposures. The computational efficiency of such an approach would allow a large portfolio to be recalculated daily and allow the computation of credit expected loss and credit Value-at-Risk and stress tests.

# References


Altman, E. (1968), " Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy," Journal of Finance 23, 1968, pp. 589–609.

Benzschawei, T. and Adler, D. (2003), "Using Equity Market Information to Predict Corporate Bond Spreads," Citigroup's Quantitative Credit Analyst, 2003, pp. 5-23.

Black, F. and Scholes, M. (1973), "The Pricing of Options and Corporate Liabilities," Journal of Political Economy 81, 1973, pp. 631-634.

Crosbie, P. (1999), "Modeling Default Risk," KMV LLC, San Francisco, January 12, 1999, pp. 1-33.

Crouhy, M., Galai, D., Mark, R. (2000), "A Comparative Analysis of Current Credit Risk Models", Journal of Banking and Finance 24, 2000, pp. 59-117.

Duffie, D. and Singleton, K. (1999). "Modeling Term Structures of Defaultable Bonds", Review of Financial Studies 12, 1999, pp. 687—720.

Finger, Christopher C. (2002), "CreditGrades Technical Document", RiskMetrics Group 2002.

Jarrow, R. and Turnbull, S. (1995). "Pricing Derivatives on Financial Securities Subject to Credit Risk," The Journal of Finance 50, 1995, pp. 53–85.

Merton, R.C. (1974), "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," Journal of Finance 29, 1974, pp. 449-470.