Measuring Skewness and Kurtosis

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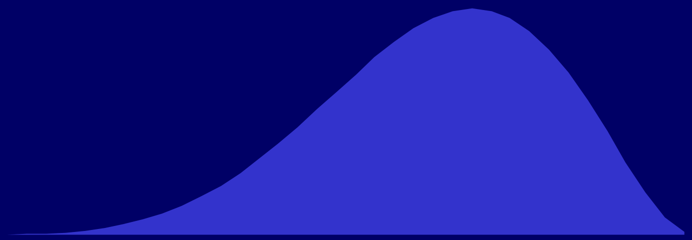
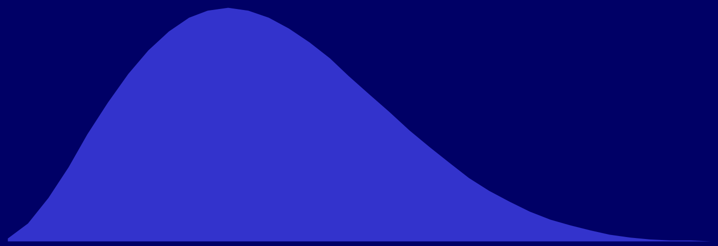
Why are Returns Skewed?



Portfolio containing short put options



Portfolio containing long call options

Defining Skewness

Skewness is the standardized 3rd central moment of a distribution

$$s = \frac{E(X - \mu)^3}{\sigma^3}$$

- Positive skewness indicates a long right tail
- Negative skewness indicates a long left tail
- Zero skewness indicates a symmetry around the mean

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Calculating Skewness

Given a set of returns r_t, $t = 1,2,\ldots T$

$$\hat{s} = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_t - \bar{r}}{\hat{\sigma}}\right)^3$$

Where $\bar{r}$ and $\hat{\sigma}$ are the estimated average and standard deviation

$$\bar{r} = \frac{1}{T}\sum_{t=1}^{T} r_t$$

$$\hat{\sigma} = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(r_t - \bar{r})^2}$$

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Skewness Adjustment

- A gamma distribution is a better proxy for skewed portfolios

Skewness	# of σ's necessary to achieve 99%
-2.83	3.99
-2.00	3.61
-1.00	3.03
-0.67	2.80
0.00	**2.33**
0.67	1.83
1.00	1.59
2.00	0.99
2.83	0.71

← **Symmetric (Normal Distribution)**

Where $\hat{\sigma}$ is the standard deviation

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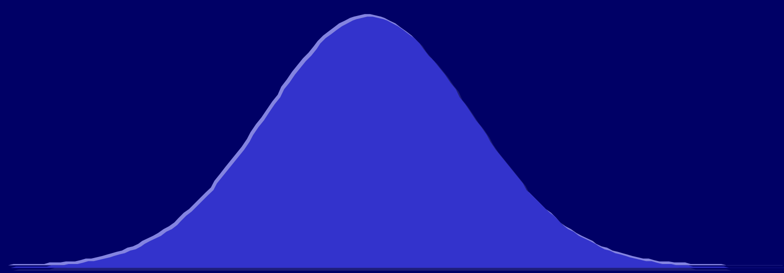
Why are Returns Kurtotic?

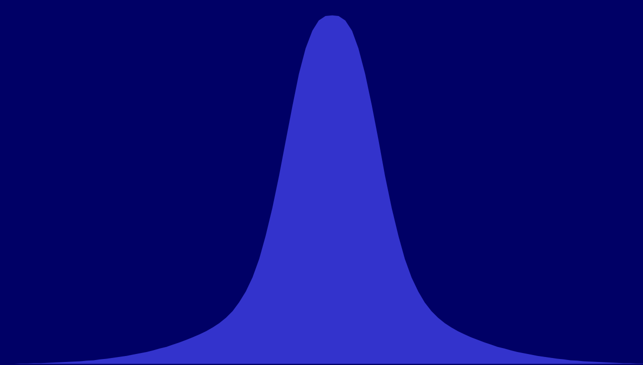
- Frequent medium to large changes
- Less frequent very large changes



Positive excess kurtosis

(internet stocks)

- Very frequent small changes
- Less frequent very large changes



Negative excess kurtosis

(Pegged exchange rate)

Defining Kurtosis

Kurtosis is the standardized 4th central moment of a distribution

$$\kappa = \frac{E(X - \mu)^4}{\sigma^4}$$

$$excess\_\kappa = \kappa - 3$$

- The kurtosis for the normal distribution is 3
- Positive excess kurtosis indicates flatness (long, fat tails)
- Negative excess kurtosis indicates peakedness

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Calculating Kurtosis

Observation of a set of returns : $r_t, \quad t = 1, 2, \cdots T$.

$$\hat{\kappa} = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_t - \bar{r}}{\hat{\sigma}}\right)^4$$

where $\bar{r}$ and $\hat{\sigma}$ are the estimated average and standard deviation

$$\bar{r} = \frac{1}{T}\sum_{t=1}^{T} r_t$$

$$\hat{\sigma} = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(r_t - \bar{r})^2}$$

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