

# VaR

# Monte Carlo Simulation

Capital Market Risk Advisors



# Monte Carlo Simulation

**Monte Carlo is most helpful when some or all assets in a portfolio are not amenable to analytical treatment**

- 1 Scenario Generation -produce a large number of future price scenarios**
- 2 Portfolio valuation - for each scenario, compute a portfolio value**
- 3 Summary - report the results of the simulation, either as a portfolio distribution or as a particular risk measure**



# Monte Carlo Simulation

## Scenario generation

Monte Carlo begins with the generation of  $n$  normal variables with unit variance and correlation matrix  $\Sigma$ .

- ◆ Decompose the correlation matrix  $\Sigma$  using the Cholesky factorization, yielding  $\Sigma = A^T A$
- ◆ Generate an  $n \times 1$  vector  $Z$  of independent standard normal variables
- ◆ Let  $Y = AZ$ . The elements of  $Y$  will each have unit variance with the correlation matrix

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## Cholesky Decomposition

- ◆ Suppose  $\Sigma = A^T A$ , where  $A$  is an upper triangular matrix, how do we find  $A$ ?

$$\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ 0 & a_{22} & a_{32} \\ 0 & 0 & a_{33} \end{bmatrix}$$

$$\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} = \begin{bmatrix} a_{11}^2 & a_{11}a_{21} & a_{11}a_{31} \\ a_{11}a_{21} & a_{21}^2 + a_{22}^2 & a_{21}a_{31} + a_{32}a_{22} \\ a_{11}a_{31} & a_{21}a_{31} + a_{32}a_{22} & a_{31}^2 + a_{32}^2 + a_{33}^2 \end{bmatrix}$$

# General Result

$$a_{ii} = \left( s_{ii} - \sum_{k=1}^{i-1} a_{ik}^2 \right)^{1/2}$$

$$a_{ij} = \frac{1}{a_{ii}} \left( s_{ij} - \sum_{k=1}^{i-1} a_{ik} a_{jk} \right)^{1/2} \quad j = i+1, i+2, \dots, N$$