Variance Covariance Implementation

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Variance Covariance P&L Formula

$$\text{VAR} = \sqrt{\sum_{i=1}^{N} \text{delta}_i^2 \sigma_i^2 + \sum_{\substack{i=1 \\ i \neq j}}^{N} \sum_{j=1}^{N} \text{delta}_i \text{delta}_j \sigma_i \sigma_j \rho_{i,j}}$$

Position Inputs

Volatility Inputs

Volatility & Correlation Inputs

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Variance Covariance Formula Components

Variance Term

$$\sum_{i=1}^{N} \text{delta}_i^2 \sigma_i^2$$

Covariance Term

$$+ \sum_{\substack{i=1 \\ i \neq j}}^{N} \sum_{j=1}^{N} \text{delta}_i \text{delta}_j \overbrace{\sigma_i \sigma_j \rho_{i,j}}$$

The first term multiplies the position by the change in price/rate – but it squares the terms. This calculates the P&L of the positions as if the assets were independent (zero correlation).

The second term then adds or subtracts an amount to adjust for the diversification effect. If the correlation is negative, the second term reduces the P&L and thus the risk. If the correlation is positive, the second term increases the risk (assuming positive/long delta positions).

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Variance Covariance
Mathematical Theory

Summation Expression	# Assets	Fully Expanded Expression
$\sqrt{\sum_{i=1}^{I}\omega_i^2\sigma_i^2 + \sum_{\substack{i=1\\ i\neq j}}^{I}\ \sum_{j=1}^{J}\omega_i\sigma_i\omega_j\sigma_j\rho_{i,j}}$	**1**	$\sqrt{\omega^2\sigma^2}$
	2	$\sqrt{\omega_i^2\sigma_i^2 + \omega_j^2\sigma_j^2 + 2\left(\omega_i\sigma_i\omega_j\sigma_j\rho_{i,j}\right)}$
	3	$\sqrt{\begin{array}{l}\omega_i^2\sigma_i^2 + \omega_k^2\sigma_k^2 + \omega_j^2\sigma_j^2 + \\ 2\left(\omega_i\sigma_i\omega_j\sigma_j\rho_{i,j} + \omega_i\sigma_i\omega_k\sigma_k\rho_{i,k} + \omega_k\sigma_k\omega_j\sigma_j\rho_{k,j}\right)\end{array}}$

$\omega_i = \text{Delta}_i$

$\sigma_i = \text{Volatility}_i$

$\rho_{i,j} = \text{Correlation}_{i,j}$

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Variance Covariance
Linear Algebra Implementation

- To simplify the calculation process, the summation formula can be transformed into a linear equation.

$$\sqrt{\sum_{i=1}^{N} \text{delta}_i^2 \sigma_i^2 + \sum_{\substack{i=1 \\ i \neq j}}^{N} \sum_{j=1}^{N} \text{delta}_i \text{delta}_j \sigma_i \sigma_j \rho_{i,j}} \equiv \sqrt{\vec{V}\, \Sigma \vec{V}^T}$$

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Variance Covariance
Linear Algebra Implementation

- The linear equation contains only a simple risk vector and the correlation matrix.

$$\sqrt{\vec{V}\,\Sigma\,\vec{V}^{T}}$$

Simple Risk Vector

Correlation Matrix

Simple Risk Vector Transposed

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Variance Covariance
Simple Risk Vector V

- The simple risk vector (V) is the product of the position vector times the volatility vector.

Position Vector **Simple Risk Vector *V***

$$\begin{pmatrix} delta_1 \\ delta_2 \\ delta_3 \\ delta_4 \\ delta_5 \\ delta_6 \end{pmatrix} \times \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{pmatrix} = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{pmatrix}$$

Volatility Vector

Example:

$$\begin{pmatrix} 180{,}000 \\ 7{,}000 \\ -1{,}125 \end{pmatrix} \times \begin{pmatrix} .10 \\ 10 \\ 6 \end{pmatrix} = \begin{pmatrix} 18{,}000 \\ 70{,}000 \\ -6{,}750 \end{pmatrix}$$

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Variance Covariance Matrix Math

- The simple risk vector (V) is then multiplied by the correlation matrix S and the vector V transposed.

Correlation Matrix Σ

$$\left[\begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{pmatrix} \times \begin{pmatrix} 1 & \rho_{1,2} & \rho_{1,3} & \rho_{1,4} & \rho_{1,5} & \rho_{1,6} \\ \rho_{2,1} & 1 & \rho_{2,3} & \rho_{2,4} & \rho_{2,5} & \rho_{2,6} \\ \rho_{3,1} & \rho_{3,2} & 1 & \rho_{3,4} & \rho_{3,5} & \rho_{3,6} \\ \rho_{4,1} & \rho_{4,2} & \rho_{4,3} & 1 & \rho_{4,5} & \rho_{4,6} \\ \rho_{5,1} & \rho_{5,2} & \rho_{5,3} & \rho_{5,4} & 1 & \rho_{5,6} \\ \rho_{6,1} & \rho_{6,2} & \rho_{6,3} & \rho_{6,4} & \rho_{6,5} & 1 \end{pmatrix} \times \begin{pmatrix} V_1 & V_2 & V_3 & V_4 & V_5 & V_6 \end{pmatrix}\right]^{\frac{1}{2}} = VAR$$

Simple Risk Vector *V* Transposed

Simple Risk Vector *V*

Variance Covariance VAR Result

$$\sqrt{\begin{pmatrix} 18{,}000 \\ 70{,}000 \\ -6{,}750 \end{pmatrix} \times \begin{pmatrix} 1 & .4 & .6 \\ .4 & 1 & .5 \\ .6 & .5 & 1 \end{pmatrix} \times \begin{pmatrix} 18{,}000 & 70{,}000 & -6{,}750 \end{pmatrix}} = \$83{,}041$$

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